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A local-global question in automorphic forms. (English summary)
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Let $H \subset G$ be reductive linear groups over a number field F such that H contains the center Z of G. Let $\mathbb{A}$ denote the adèle ring of F. An automorphic representation $\Pi \subset L^2(G(F)Z(\mathbb{A})\backslash G(\mathbb{A}))$ is said to be *H-distinguished*, if there exists $\phi \in \Pi$ such that the period integral

$$P(\phi) = \int_{H(F)Z(\mathbb{A})\backslash H(\mathbb{A})} \phi(x)dx$$

is nonzero. A local representation Π_v of $G(F_v)$ is called distinguished, if there exists an $H(F_v)$-invariant linear form on Π_v. The present paper is concerned with the following two local-global questions:

(1) If $\Pi = \bigotimes_v \Pi_v$ is cuspidal such that each local component Π_v is distinguished, is there an automorphic representation Π' in the same L-packet as Π, which is globally distinguished?
(2) If $\Pi = \bigotimes_v \Pi_v$ is globally distinguished and if $\Pi' = \bigotimes \Pi'_v$ is in the same L-packet as Π, is then Π' globally distinguished?

In the present paper, these questions are answered affirmatively in two special cases where $G = \mathrm{SL}_2$. The results given are SL_2-versions of older GL_2-results which are due to Harder-Langlands-Rapoport in one case and Waldspurger in the other. Another related question, the answer of which in the present cases is used for the proofs, is how to determine whether a given representation is distinguished or not in terms of the Langlands parameters. *Anton Deitmar*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.