

Roll No. :

Branch :BTech/Dual//CL/EE/EP

Tutorial Room :

Verion:I

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
Department of Mathematics

MA 205 - Comp. Anal. Quiz

Tuesday 22nd August 2012

Weightage: 10 marks

Duration: 45 minutes

In Questions No 1-4, of the four alternatives, exactly one is true. Encircle it. In Questions 5-10, fill in the blanks to obtain a mathematically correct statement. Each question carries 1 mark and there is no partial marking. You will be given separate sheets for rough work. Use the question-cum-answer sheet only for your answers.

Q.1 Let f be a non constant function which is holomorphic throughout the **upper-half plane** $\{x + iy : y > 0\}$. Which of the following is holomorphic in the **upper-half plane**?

(A) $f(-z)$ (B) $f(-iz)$ (C) $(f(z))^2$ (D) $f(iz)$. Answer(C)

Q.2 Counterclockwise rotation around $(2, 0)$ through $\pi/2$ followed by a clockwise rotation around $(0, 2)$ through $\pi/2$ is

(A) a translation by $4i$; (B) a translation by -4 ;
(C) a translation by $-4i$; (D) a translation by 4 . Answer (B)

Q.3 For a holomorphic function f on \mathbb{C} such that $K = \int_{|z|=1} \overline{f(z)} f'(z) dz$, it is given that K takes one of the following four values listed below. Then K is actually equal to

(A) 1; (B) $1 + i$; (C) $1 - i$; (D) i . Answer (D)

Q.4 Let $U = \{z = x + iy \in \mathbb{C} : |x - 2| \geq 1\}$. Then U is

- (A) open but not path connected;
- (B) path connected but not open;
- (C) path connected and open;
- (D) neither path connected nor open.

Answer (D)

Q.5 The value of the integral $\int_{|z|=5} \frac{z^2 - z + 100}{z - 1} dz$ is equal to

$200\pi i$.

Q.6 Let f_j be the counterclockwise rotation through an angle $\pi/2$ about the point $z_j, j = 1, 2, 3$ respectively. Then the function $f_3 \circ f_2 \circ f_1$ is a rotation about the point

$iz_1 + z_2 - iz_3$.

Q.7 Let $f(x, y) = u(x, y) + v(x, y)$, where $u(x, y) = 5x^2 + 2xy$; $v(x, y) = x^2 + 2xy - 2y^2 - 4y$. Then the set of all points (a, b) at which f satisfies Cauchy-Riemann equations is

$\{(1, -2)\}$

Q.8 A harmonic conjugate of the function $u(x, y) = \sinh x \sin y$ is given by

$-\cosh x \cos y + c$.

Q.9 If $a + ib = i^{i+1}$, then the value set of b is

$\{e^{(2n-1/2)\pi} : n \in \mathbb{Z}\}$.

Q.10 The integral of the function $f(x, y) = x^3$ along the curve $y = 1 + x + \dots + x^{100}$ from $x = 0$ to $x = 2$ is

$4 + i\left(\sum_{j=1}^{100} \frac{j2^{j+3}}{j+3}\right)$

Best of Luck