Roll No. : **Branch :BTech/Dual//CL/EE/EP**

Tutorial Room : Version:II

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
Department of Mathematics

MA 205 - Comp. Anal. Quiz

Tuesday 22nd August 2012 **Weightage: 10 marks**

Duration: 45 minutes

In Questions No 1-4, of the four alternatives, exactly one is true. Encircle it. In Questions 5-10, fill in the blanks to obtain a mathematically correct statement. Each question carries 1 mark and there is no partial marking. You will be given separate sheets for rough work. Use the question-cum-answer sheet only for your answers.

Q.1 Let f be a non constant function which is holomorphic throughout the **right-half plane** $\{x + \imath y \ : \ x < 0\}$. Which of the following is holomorphic in the **upper-half plane**?

(A) $f(-z)$ (B) $f(-\imath z)$ (C) $(f(z))^2$ (D) $f(\imath z)$. Answer (D)

Q.2 (II) Counterclockwise rotation around $(0, 2)$ through $\pi/2$ followed by a clockwise rotation around $(2, 0)$ through $\pi/2$ is

(A) a translation by $4\imath$; (B) a translation by -4;

(C) a translation by $-4\imath$; (D) a translation by 4. Answer (D)

Q.3 For a holomorphic function f on $\mathbb{C}$ such that $K = \imath \int_{|z|=1} \overline{f(z)} f'(z) dz$, it is given that K takes one of the following four values listed below. Then K is actually equal to

(A) 1; (B) $1 + \imath$; (C) $1 - \imath$; (D) $\imath$. Answer (A)

Q.4 Let $U = \{z = x + \imath y \in \mathbb{C} \ : \ |x - 2| \leq 1\}$. Then U is

(A) open but not path connected;

(B) path connected but not open;

(C) path connected and open;

(D) neither path connected nor open. Answer (B)

Q.5 The value of the integral $\int_{|z|=5} \dfrac{z^2 - z + 101}{z - 1} dz$ is equal to

$\underline{202\pi\imath.}$

Q.6 Let f_j be the counterclockwise rotation through an angle $\pi/2$ about the point $z_j, j = 1, 2, 3$ respectively. Then the function $f_1 \circ f_2 \circ f_3$ is a rotation about the point

$\underline{\imath z_3 + z_2 - \imath z_1.}$

Q.7 Let $f(x, y) = u(x, y) + \imath v(x, y)$, where $u(x, y) = 5x^2 + 2xy$; $v(x, y) = x^2 + 2xy - 2y^2 - 8y$. Then the set of all points (a, b) at which f satisfies Cauchy-Riemann equations is

$\underline{\{(2, -4)\}.}$

Q.8 A harmonic conjugate of the function $u(x, y) = \sinh x \sin y$ is given by

$\underline{-\cosh x \cos y + c.}$

Q.9 If $a + \imath b = \imath^{\imath - 1}$, then the value set of b is

$\underline{\{-e^{(2n - 1/2)\pi} \ : \ n \in \mathbb{Z}\}.}$

Q.10 The integral of the function $f(x, y) = x^3$ along the curve $y = 1 + x + \cdots + x^{100}$ from $x = 0$ to $x = 3$ s

$\underline{3^4/4 + \imath(\sum_{j=1}^{100} \frac{j3^{j+3}}{j+3})}$

Best of Luck