

Roll No. :

Branch :BTech/Dual//CL/EE/EP

Tutorial Room :

Version:III

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY  
Department of Mathematics

MA 205 - Comp. Anal. Quiz

Tuesday 22nd August 2012

Weightage: 10 marks

Duration: 45 minutes

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In Questions No 1-4, of the four alternatives, exactly one is true. Encircle it. In Questions 5-10, fill in the blanks to obtain a mathematically correct statement. Each question carries 1 mark and there is no partial marking. You will be given separate sheets for rough work. Use the question-cum-answer sheet only for your answers.

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Q.1 Let  $f$  be a non constant function which is holomorphic throughout the **lower-half plane**  $\{x + iy : y < 0\}$ . Which of the following is holomorphic in the **upper-half plane**?

(A)  $f(-z)$       (B)  $f(-iz)$       (C)  $(f(z))^2$       (D)  $f(iz)$ . Answer (A)

Q.2 Clockwise rotation around  $(2, 0)$  through  $\pi/2$  followed by a counter-clockwise rotation around  $(0, 2)$  through  $\pi/2$  is

(A) a translation by  $4i$ ;                      (B) a translation by  $-4$ ;  
(C) a translation by  $-4i$ ;                      (D) a translation by  $4$ . Answer (A)

Q.3 For a holomorphic function  $f$  on  $\mathbb{C}$  such that  $K = (1+i) \int_{|z|=1} \overline{f(z)} f'(z) dz$ , it is given that  $K$  takes one of the following four values listed below.

Then  $K$  is actually equal to

(A) 1;    (B)  $1 + i$ ;    (C)  $1 - i$ ;    (D)  $i$ .                      Answer (C)

Q.4 Let  $U = \{z = x + iy \in \mathbb{C} : |x - 2| < 1\}$ . Then  $U$  is

- (A) open but not path connected;
- (B) path connected but not open;
- (C) path connected and open;
- (D) neither path connected nor open.

Answer (C)

Q.5 The value of the integral  $\int_{|z|=5} \frac{z^2 - z + 102}{z - 1} dz$  is equal to

$204\pi i$ .

Q.6 Let  $f_j$  be the counterclockwise rotation through an angle  $\pi/2$  about the point  $z_j, j = 1, 2, 3$  respectively. Then the function  $f_1 \circ f_3 \circ f_2$  is a rotation about the point

$iz_2 + z_3 - iz_1$ .

Q.7 Let  $f(x, y) = u(x, y) + v(x, y)$ , where  $u(x, y) = 5x^2 + 2xy$ ;  $v(x, y) = x^2 + 2xy - 2y^2 - 12y$ . Then the set of all points  $(a, b)$  at which  $f$  satisfies Cauchy-Riemann equations is

$\{(3, -6)\}$

Q.8 A harmonic conjugate of the function  $u(x, y) = \sinh x \sin y$  is given by

$-\cosh x \cos y + c$ .

Q.9 If  $a + ib = i^{i+1}$ , then the value set of  $b$  is  $\{e^{-(2n+1/2)\pi} : n \in \mathbb{Z}\}$

Q.10 The integral of the function  $f(x, y) = x^3$  along the curve

$y = 1 + x + \dots + x^{100}$  from  $x = 0$  to  $x = 4$  is  
 $4^3 + i(\sum_{j=1}^{100} \frac{j4^{j+3}}{j+3})$

Best of Luck