# International Workshop/Conference in Geometry and Topology
# Golden Jubilee Meeting

*17th to 26th August 2007*

***Department of Mathematics***

***Indian Institute of Technology, Bombay***

## Final Report

CONTENTS

## 1. Organizing Committee

S. R. Ghorpade, IITB
R. S. Kulkarni, IITB
Akhil Ranjan, IITB
Parameswaran Sankaran IMSc Chennai (Academic Adviser)
Peter Zvengrowski, Uni. of Calgary, Canada (Academic Adviser)
Anant R. Shastri, IITB (Convener)
email:ars@math.iitb.ac.in

## 2. Preamble

On the occasion of the **Golden Jubilee Year** of Indian Institute of Technology Bombay, the Department of Mathematics is organizing year-long programme during June 2007 to May 2008.

As part of this activity, an International workshop/conference in Geometry and Topology was held during the above dates. The theme of the workshop was interaction between and Differential Geometry and Topology. Eminent scholars from abroad and within the country spared their valuable time in participating in this program. Research scholars all over the country attended both workshop as well as the conference. We have been able to put-up a program of academically excellent quality with a cohesive theme, thanks to initiative shown by the two academic advisers and the ready response that we got from so many experts.

Right on the first day, all participants received a bound copy the entire lecture notes for the 5-day workshop. Lecture notes of lectures delivered in the conference and other relevant reading materials were made available on demand and supply.

There were three plenary talks one on each day of the conference. Each of them have been video recorded and all participants were given a copy of the same on three DVDs This experimental move was welcome with a lot of enthusiasm by the participants.

Each evening, there were some cultural activities supported by some professional musicians. In one of them, all delegates could take part and some of them came forward to share these lighter moments with the rest of the delegates. It should be mentioned that the evening with Pandit Pushpraj Koshti on SURBAHAR was appreciated very much by ou foreign delegates.

The program was generously supported by NBHM and DST. We have to act on several things even before we got the word of support from any of them. The Institute authorities came forward here with generous grant of seed money. The organizers are thankful to them and the two funding agencies.

## 3. List of Outside Participants

(C)= only in the conference.

| Sr. no. | Name | Affiliation | Arrival | Departure | Remark |
|---|---|---|---|---|---|
| 1. | K.Y. Lam | Canada | 15th Aug | 27th August | |
| 2. | Peter Wong | USA | 14th August | 26th Aug. | |
| 3. | Peter Zvengrowski | Canada | 16th Aug | 27th Aug | |
| 4. | K. Varadarajan | Canada | 15th Aug | 26th Aug. | |
| 5. | S. M. Srivastava | ISI Kol | 16th Aug. | 26th Aug. | |
| 6. | Goutam Mukherjee | —”’—- | —-””— | —””—- | |
| 7. | Amiya Mukherjee | —”’—- | —-””— | —””—- | |
| 8. | P Sankaran | IMSc | 17th Aug. | 25th Aug. | |
| 9. | Himadri Mukherjee | NEHU | 22nd Aug. | 26th Aug. | (C) |
| 10 . | Basudeb Datta | IISc | 22nd Aug | 25th Aug | (C) |
| 11. | C. S. Aravinda | TIFR Bang. | 23rd Aug. | 25th Aug | (C) |
| 12 | M. S. Narasimhan | TIFR Bang | 22nd Aug | 26th Aug | (C) |
| 13. | S. Ramanan | CMI | 23rd Aug. | 26th Aug. | (C) |
| 14. | R. V. Gurjar | TIFR | 24th Aug. | 25th Aug. | (C) |
| 15 | M. S. Raghunathan | TIFR | 23rd Aug | 25th Aug. | (C) |
| 16 | Nitin Nitsure | TIFR | 25th Aug. | 25th Aug. | (C) |
| 17 | Jaya Iyer | IMSc | 22nd Aug | 26th Aug | (C) |
| 18 | S. P. Tripati | KMC, Delhi | 15th Aug. | 26th Aug. | |
| 19. | Swagatha Sarkar | IMSc. | 16th Aug. | 26th Aug. | |
| 20. | Shilpa Gondhali | TIFR | 16th Aug. | 26th Aug. | |
| 21 | Sam Johnson | Chennai | 16th Aug. | 26th Aug. | |
| 22 | Mahendra Singh | HRI | 16th Aug. | 27th Aug | |
| 23 | Sankar Raj | IITM | 16th Aug. | 26th Aug. | |
| 24 | S. Balaji | Chennai | 16th Aug. | 26th Aug. | |
| 25 | Biswajit Ranasingh | NIT Rourkel | 16th Aug. | 26th Aug. | |
| 26 | Niraj Prasad | Mumbai Uni | 16th Aug. | 26th Aug. | |
| 27 | Ajay Thakur | IMSc | 16th Aug. | 26th Aug. | |
| 28 | Anisa Mohamad | IMSC | 16th Aug. | 26th Aug. | |

## 4. List of Participants from the Institute

1. Prof. R. S. Kulkarni
2. Prof. Akhil Ranjan
3. Prof S. R. Ghorpade
4. Prof A. R. Shastri
5. Prof. U. K. Anandavardhanan
6. Prof. T. J. Puthenpurakkal
7. Prof. J. K. Verma
8. Prof. Ravi Raghunathan
9. Prof. Manoj Keshari
10. Prof. D. V. Pai
11. Prof. Preeti Raman
12. Prof. M. K. Srinivasan
13. Dr. Vinay Wagh
14. Mr. B. Subhash
15. Mr. Krishnendu Gangopadhyay
16. Mr. Vikram Aithal

## 5. List of Experts in the Workshop

| | | |
|---|---|---|
| Prof. K. Y. Lam | University of British Columbia, | Canada |
| Prof K. Varadarajan | University of Calgary | Canada |
| Prof. P. Wong | Bates College, Maine | USA |
| Prof. P. Zvengrowski | University of Calgary | Canada |

## 6. List of Speakers in the Conference

| | | |
|---|---|---|
| Prof. K. Y. Lam | University of British Columbia, | Canada |
| Prof K. Varadarajan | University of Calgary | Canada |
| Prof. P. Wong | Bates College, Maine | USA |
| Prof. P. Zvengrowski | University of Calgary | Canada |
| Prof. Goutam Mukherjee | ISI Kolkata | India |
| Prof. Amiya Mukherjee | ISI Kolkata | India |
| Prof. P Sankaran | IMSc Chennai | India |
| Prof. Himadri Mukherjee | NEHU Shillong | India |
| Prof. Basudeb Datta | IISc Bangalore | India |
| Prof. C. S. Aravinda | TIFR Bangalore center | India |
| Prof. M. S. Narasimhan | TIFR Bangalore Center | India |
| Prof. S. Ramanan | CMI Chennai | India) |
| Prof. R. V. Gurjar | TIFR Mumbai | India |
| Prof. M. S. Raghunathan | TIFR, Mumbai | India |
| Prof. Nitin Nitsure | TIFR, Mumbai | India |
| Prof. R. S. Kulkarni | IIT Bomaby | India |
| Dr. Jaya Iyer | IMSc Chennai | India |
| Ms Swagatha Sarkar | IMSc. Chennai | India |
| Mr. B Subhash | IIT Bombay | India |

## 7. Schedule of Lectures in the Workshop

| Date | 0930-1100 | | 1130-1300 | | 1430-1600 | | 1630-1800 |
|---|---|---|---|---|---|---|---|
| 17/8 (Friday) | KYL | T | KV | L | PW | T | PZ |
| 18/8 (Saturday) | KYL | E | KV | U | PW | E | PZ |
| 19/8 (Sunday) | KYL | A | KV | N | PW | A | PZ |
| 20/8 (Monday) | KYL | TI | KV | C | PW | TI | PZ |
| 21/8 (Tuesday) | KYL | ME | KV | H | PW | ME | PZ |
| 22/8 Wednesday) | FREE | | FREE | | FREE | | FREE |

# Speakers and titles

**1. Prof. K. Y. Lam (KYL)** *Topological methods in the study of bilinear forms*
**2. Prof. K. Varadarajan (KV)** *Wall obstruction*
**3. Prof. Peter Wong (PW)** *Geometric and combinatorial group theoretic methods in fixed point theory*
**4. Prof. Peter Zvengrowski (PZ)** *On Seifert Manifolds*

# 8. Schedule of talks in the Conference: 23rd-25th August 2007

| | Thursday | Friday | Saturday |
|---|---|---|---|
| Dates | 23/8/07 | 24/8/07 | 25/8/07 |
| **Chairman** | **Peter Zvengrowski** | **Peter Wong** | **K Varadarajan** |
| 09:30-10:25 | M. S. Narasimhan | S Ramanan | M. S. Raghunathan |
| 10:30-11:15 | A. Mukherjee | R. V. Gurjar | P. Zvengrowski |
| 11:15-11:30 | TEA BREAK | | |
| 11:30-12:15 | B. Datta | G. Mukherjee | N. Nitsure |
| 12:20-13:05 | P. Wong | C. S. Aravinda | Jaya Iyer |
| 13:05-14:30 | LUNCH BREAK | | |
| **Chairman** | **K. Y. Lam** | **P. Sankaran** | **A. Mukherjee** |
| 14:30-15:15 | Swagata Sarkar | B. Subhash | K. Y. Lam(2) |
| 15:15-15:40 | TEA BREAK | | |
| 15:40-16:25 | K. Varadarajan | K. Y.Lam(1) | Anisa Mohammad |
| 16:30-17:15 | P. Sankaran | H. Mukherjee | R. S. Kulkarni |
| 17:15–18:00 | SNACK TIME | | |

# 9. Titles of the Conference Talks

| **Speaker** | | **Topic** |
|---|---|---|
| 1. M. S. Narasimhan | : | Geometry and Physics (Plenary) |
| 2. K. Y. Lam | : | (1) Vector bundles of low geometric dimension |
| | : | over real projective spaces |
| 3. K. Y. Lam | : | (2) The Yuzvinsky conjecture arising from sums of squares |
| | : | and its partial solution |
| 4. M. S. Raghunathan | : | Bruhat-Tits theory - an overview (Plenary) |
| 5. Nitin Nitsure | : | Sign lemma for dimension shifting |
| 6. Swagata Sarkar | : | Degrees of maps between Grassmannians |
| 7. Basudeb Datta | : | Minimal triangulations of sphere bundles over the circle |
| 8. Goutam Mukherjee | : | Deformation of Leibniz Algebras |
| 9. C. S. Aravinda | : | Ricci flow and negative curvature. |
| 10. P. Wong | : | Fox homotopy groups, Gottlieb groups, Rhodes groups, |
| | : | and generalized Whitehead products |
| 11. B. Subash | : | Linear Morse Functions |
| 12. Kalathoor Varadarajan | : | Anti-hopfian and anti co-hopfian modules. |
| 13. R. S. Kulkarni | : | Dynamics of Linear and Affine Maps. |
| 14. Amiya Mukherjee | : | Rational Pontryagin classes and foliations |
| 15. Himadri Mukherjee | : | On problems of classification of manifolds |
| 16. Parameswaran Sankaran | : | K theory of torus manifolds |
| 17. R. V. Gurjar | : | On the Shafarevich Conjecture for genus-2 fibrations |
| 18. Peter Zvengrowski | : | Group Cohomology of Finite Fundamental Groups |
| | : | of 3-Manifolds |
| 19. Jaya Iyer | : | Chern-Simons classes of canonical extensions of flat bundles |
| 20. S. Ramanan | : | Interaction between Geometry and Topology over the years. (Plenary) |

# 10. ABSTRACTS OF THE WORKSHOP

**1. Topological Methods in the Study of Bilinear Forms** *Kee Yuen Lam*

| | |
|---|---|
| Talk 1 | Motivation: Examples of nonsingular bilinear maps |
| | (classical, Hurwitz-Radon and polynomial maps) |
| | The main question |
| | The Stiefel-Hopf Condition for the existence of a nonsingular |
| bilinear map | $F : R^r \times R^s \to R^n$ |
| Talk 2 | Behrend's proof of the Stiefel-Hopf Condition |
| | Further constructions of nonsingular bilinear maps |
| | A linearization procedure |
| | Cohomology proof of the Stiefel-Hopf Condition |
| Talk 3 | The topology of projective spaces and truncated projective spaces |
| | The equal height case |
| | Further necessary conditions for existence of nonsingular bilinear maps |
| | A classical problem about sums of squares |
| Talk 4 | Algebraic analysis of sums of squares identities |
| | Geometric analysis of sums of squares identities |
| | Relationship with minimal submanifolds of spheres |
| Talk 5 | Sums of squares identities with integer coefficients |
| | relationship with matrix colorings |
| | The Yuzvinsky conjecture on matrix coloring |
| | Proof of some cases of Yuzvinsky's conjecture |
| | Open problems |

**2. Wall Obstrcuction:** *Speaker. K. Varadarajan*

All the spaces we consider will be assumed to be $0$-connected.We say that $X$ is dominated by $Y$ in homotopy if there exist maps $f : X \to Y$ and $g : Y \to X$ with $g \circ f$ homotopic to $Id_X$. A result of J.H.C.Whitehead asserts that any space dominated by a CW-complex is of the homotopy type of a CW-complex.When X is dominated by a finite complex ,whether $X$ is of the homotopy type of a finite complex remained unsolved for sometime. Whitehead regarded this problem to be a very hard problem.In 1965, C.T.C.Wall gave a negative solution to this problem. Let X be a finitely dominated space, $\pi = \pi_1(X)$ and $(\tilde{K}_0)(Z(\pi))$ the reduced projective class group of the integral group ring $Z(\pi)$ of $\pi$.C.T.C.Wall associated with $X$ an element $\tilde{w}(X)$ in $\tilde{K}_0)(Z(\pi))$ depending only on the homotopy type of $X$, whose vanishing was proved to be necessary and sufficient for X to be of the homotopy type of a finite complex.Given any finitely presented group $\pi$ and any u in $(\tilde{K}_0)(Z(\pi))$, C.T.C.Wall constructed a finitely dominated space $X$ with $\tilde{w}(X) = u$. Using Dock-Sang Rim's results and the non triviality of the class group $Cl(Z[w])$ when $w = exp(2\pi i/23)$, C.T.C.Wall finds a negative solution to the problem of Whitehead. While dealing with geometric problems in Differential Topology, Dennis sullivan developed the technique of localising spaces at a given set of primes. A systematic account of localising nilpotent groups and nilpotent spaces is given in the treatise by P.Hilton. G.Mislin and J.Roitberg.Later G.Mislin started studying restrictions satisfied by the Wall obstructions of finitely dominated nilpotent spaces.I have some contributions individually and jointly with G.Mislin in this study. My lectures in the work shop will mention some of these results. Lately I have been working on different problems. Still I might mention some open problems which to me appear very challenging.

**3. Combinatorial and Geometric Group Theoretic Methods in Fixed Point Theory** *Peter Wong*

Classical Nielsen-Wecken fixed point theory combines homological techniques of Lefschetz and covering space theory to give a better homotopy invariant for estimating the number of fixed points of a selfmap within its homotopy class. This invariant, now known as the Nielsen number, is often a sharp lower bound for the minimal number of fixed points but its calculation is notoriously difficult.

In a series of lectures, we survey existing techniques for computing the Nielsen number. In particular, we discuss how methods from combinatorial group theory and from geometric group theory can be applied to the study of topological fixed point theory.

**4. An Introduction to $\ell_2$-Homology:** *P. Zvengrowski*

These lectures will give an introduction to the subject of $\ell_2$-homology, a homology theory introduced in 1976, which has since grown in importance for both its applications and for its relations with other new theories. The theory will be set up in three separate stages : functional analysis (review of Hilbert space), algebra (the concept of Hilbert $G$-modules), and topology (with some emphasis on harmonic chains). Once all this is accomplished some of the applications will be discussed, including deficiency of a (finitely presented) group and amenable groups, as well as relations with other homology theories.

# 11. ABSTRACTS OF THE CONFERENCE

**1. M. S. Narasimhan** *Geometry and Physics*

I shall talk about the interaction between geometry and physics in mathematical research.Illustrations will be given of insights provided by physics in some geometric problems and of the use of sophisticated mathematics in physics.

**2. M. S. Raghunathan** *Bruhat-Tits theory - an overview*

In this talk I will briefly outline the theory of Bruhat-Tits buildings associated to simply connected algebraic groups over local fields and indicate some interesting applications of the theory.

**3. Nitin Nitsure** *Sign lemma for dimension shifting.*

There is a surprising occurrence of some minus signs in the isomorphisms produced in the well-known technique of dimension shifting in calculating derived functors in homological algebra. We explicitly determine these signs. Getting these signs right is important in order to avoid basic contradictions. We illustrate the lemma by some de Rham cohomology and Chern class considerations for compact Riemann surfaces.

**4. Swagata Sarkar** *Degrees of maps between Grassmannians*

In this talk, we will discuss the possible degrees of continuous maps between two complex Grassmann manifolds of the same dimension. In particular, we will prove the following theorem:

Let $G_{n,k}$ denote the complex Grassmann manifold of $k$-dimensional vector subspaces of $C^n$. Let $2 \leq l < k$ be arbitrary positive integers. Let $f : G_{n,k} \longrightarrow G_{m,l}$ be *any* continuous map between complex Grassmann manifolds of the same dimension. Then, there exists a positive integer $N$ such that $deg(f) = 0$ whenever $n, m \geq N$.

**5. Basudeb Datta** *'Minimal triangulations of sphere bundles over the circle'*

In 1986, Kuhnel constructed a $(2d + 3)$-vertex $d$-dimensional simplicial complex $K^d_{2d+3}$ which triangulates a sphere bundle over the circle for each $d \geq 2$. In 1987, Brehm and Kuhnel showed that any triangulation of a non-simply connected closed pl manifold of dimension $d \geq 3$ requires at least $2d + 3$ vertices.

In a recent paper, we have shown that $K^d_{2d+3}$ is the unique $(2d + 3)$-vertex triangulation of a non-simply connected closed $d$-manifold for all $d \geq 3$. In this talk, we would like to present an outline of the proof of our main result together with some constructions.

**6. P. Wong** *Fox homotopy groups, Gottlieb groups, Rhodes groups, and generalized Whitehead products*

In 1945, R. Fox introduced the torus homotopy groups in order to give a geometric interpretation of the classical Whitehead product. Using modern language, we re-interpret Fox's results and show that the generalized Whitehead products are also commutators in the torus homotopy groups. We discuss in this setting generalizations of the classical Gottlieb groups and their equivariant analogs in Rhodes groups. Jacobi identities for the generalized Whitehead products are given in a unified approach using the Fox homotopy groups.

**7. Kalathoor Varadarajan** *Anti-hopfian and anti co-hopfian modules.*

Hiramo and Mogami refer to a module $M$ as anti hopfian if $M$ is not simple and every non zero factor module $\overline{M}$ of $M$ is isomorphic to $M$. They completely characterize anti hopfian (right) R-modules $M$ over rings $R$ possessing the property that all cyclic (right) R-modules are hopfian, by properties of the lattice $L(M)$ of submodules of $M$ and use this characterization to study the endomorphism ring $S = End(M_R)$.

In our present paper we define a non zero module $M$ to be anti co-hopfian if $M$ is not simple and every non zero submodule of $M$ is isomorphic to $M$. The proofs of Hirano and Mogami depend heavily on the fact that any non-zero module $M$ admits a subdirectly irreducible (equivalently cocyclic) factor module. The dual notion to that of cocyclicity is that of a local module. In general, it is not true that every non zero module $M$ admits a local submodule. In the present paper, we completely characterize the rings $R$ satisfying the condition that every non-zero (right) $R$-module possesses a local submodule. If $R$ is such a ring further satisfying the condition that every cocyclic right $R$-module is co-hopfian, then we characterize anti co-hopfian right $R$-modules $M$ by properties of the lattice $L(M)$, dualizing results of Hirano and Mogami, and use these to obtain results on $S = End(M_R)$ for such anti co-hopfian modules.

**8. Amiya Mukherjee** *Rational Pontryagin classes and foliations*

In this talk I will outline a proof that a leafwise homotopy equivalence of compact foliated manifolds preserves the rational Pontryagin classes if one of the foliations has negatively curved leaves.

**9. Himadri Mukherjee** *On problems of classification of manifolds*

A brief survey of the classification techniques and results for higher dimensional manifolds and some openings for the lower dimensional manifolds will be given.

**10. Parameswaran Sankaran** *K theory of torus manifolds.*

The algebraic geometric notion of non-singular projective toric varieties were generalized to 'toric manifolds' (or 'quasi-toric manifolds' as they are now called) by Davis and Januskiewicz. This has

been further generalized by Masuda and Panov who introduced a new class of manifolds known as torus manifolds.

K- theory of non-singular projective toric varieties and quasi-toric manifolds have been described (in terms of generators and relations) by Sankaran and Uma. These results have been extended recently to the setting of certain torus manifolds. The talk will introduce the class of torus manifolds and describe our approach to determination of their K theory.

**11. R. V. Gurjar** *On the Shafarevich Conjecture for genus-2 fibrations.*

We will outline a proof of the following result. Theorem. Let $f : X \to C$ be a morphism on a smooth complex, projective surface $X$ onto a smooth curve $C$. Assume that a general fiber of $f$ has genus 2 and the canonical bundle $K_X$ is ample. Then the universal covering space of $X$ is holomorphically convex. This verifies a conjecture of I. Shafarevich for such X. This is a joint work with B.P. Purnaprajna.

**12. Peter Zvengrowski** *Group Cohomology of Finite Fundamental Groups of 3-Manifolds*

Thanks to the recent work of Perelman and his successors, it is now known that any 3-manifold with finite fundamental group $G$ arises from a free orthogonal action of $G$ on $S^3$. Hence it is one of the groups found by Hopf and Seifert-Threlfall, who determined all such actions around 1930. In particular the 3-manifold $M = S^3/G$ is an orientable Seifert manifold (known as a spherical space form). For orientable Seifert manifolds $M$ with infinite fundamental group $G$, the cohomology ring $H*(M; A)$ was calculated around 2000 by Bryden, Hayat, Zieschang, and the author. There are some important differences when $G$ is finite, also related to the group cohomology $H*(G; A)$ which is now 4-periodic (for $G$ infinite the group cohomology is the same as the cohomology of $M$ so vanishes in dimensions higher than 3). The finite cases, studied recently by Tomoda and the author, will be the subject of this talk.

**13. K. Y. Lam** *Vector bundles of low geometric dimension over real projective spaces*

Stably nontrivial vector bundles of low geometric dimension over real projective spaces are not easy to construct. In this talk we shall give one such construction, answering an open question posed by J.F.Adams in the positive. The main step is to make use of "KO-periodic maps" which exist between certain truncated projective spaces.

**14. Gautam Mukherjee** *Deformation of Leibniz Algebras.*

Leibniz algebras were introduced by J. L. Loday as a non-antisymmetric analogue of Lie algebras while studying periodicity phenomenon in algebraic K-theory. We studied deformation of Leibniz algebras over a commutative local algebra base and gave a construction of versal deformations Leibniz algebras. Versal deformation of a Leibniz algebra induce all other non-equivalent deformations. In the talk, I will briefly present this work.

**15. C. S. Aravinda** *Ricci Flow and Negative Curvature*

Ricci flow approach has been very fruitful in proving a good number of very satisfying results in the situation of positive curvature; most notable of them being the proof of the classical Poincare conjecture. However, some results in the negative curvature situation, specially in dimensions bigger than 2, have not been very encouraging. In this talk, I discuss a result of Farrell and Ontaneda which shows the existence of negatively curved metrics for which the Ricci flow convergence is not smooth.

**16. B. Subhash** Linear Morse Functions

Sard's theorem implies that for almost every $a \in R^n$, the function $f_a(b) = trace(ba^t),\ b \in R^n$ is a Morse function on any manifold imbedded in $R^n$. In this talk we shall consider the real manifolds namely, the Orthogonal group $O(n)$, Grassmannian $G_{nk}$ and the Stiefel manifolds $V_k(R^n)$, imbedded in $M_n(R)$ in the first two cases and $R^{nk}$ in the case of Stiefel manifold and give a charaterisation for the dense set of linear maps that are Morse functions on these manifolds.

**17. R. S. Kulkarni** *Dynamics of Linear And Affine Maps.*

Let $G$ be a group. Two elements $x, y$ of $G$ are said to be $z$-equivalent if there centralizers are conjugate. The classification of $z$-classes is an important information about the internal structure of $G$ and dynamics in any of its actions.

We shall apply this viewpoint to $GL(n)$ and $Aff(n)$.

**18. Jaya Iyer** *Chern-Simons invariants of flat bundles and their extensions.*

We will discuss the question of Cheeger-Simons on the Chern-Simons invariants of flat vector bundles on smooth manifolds. We will also relate it with some questions in the theory of algebraic cycles.

**19. Ms. Anisa Mohmad Husen Chorwadwala***Behaviour of functional associated to Boundary Value Problem over a family of domains in space forms*

# 12. COMMENTS FROM EXPERTS

## 1. Technical Report, Peter Zvengrowski

This Workshop/Conference took place at the Indian Institute of Technology, Bombay, from August 17-25, 2007, in conjunction with the Golden Jubilee of this university. As planned, it consisted of a Workshop from August 17-21, followed by a Conference from August 23-25, with a free day on August 22. The Scientific Committee consisted of Professors R. Kulkarni (USA and India), P. Sankaran (India), A.R. Shastri (Principal Organizer, India), and myself (Canada). One of the main goals was to convene a meeting that would be of value to graduate students and younger mathematicians, as well as providing a forum for experienced mathematicians and experts in the field.

I believe that the meeting was eminently successful in achieving these goals. The Workshop portion consisted of four minicourses, presented by Professors K. Y. Lam (Univ. Br. Columbia), K. Varadarajan (Univ. Calgary), P. Wong (Bates College, Maine), and myself (Univ. of Calgary). The format consisted of four 90 minute lectures daily, one for each minicourse, on each of the five days. I believe this format was very successful, each lecturer having ample time to present his material in sufficient detail. I certainly appreciated the generous time allotment in my lectures on $\ell_2$-homology, and was able to cover somewhat more material than anticipated as well as furnish more details of various proofs. Overall the material in these four minicourses encompassed a broad spectrum of important and deep parts of classical and modern algebraic topology, along with related areas in other parts of mathematics such as algebra, geometry, number theory, analysis, and applied mathematics (theory of elasticity). It was also remarkable how many interconnections appeared between the four minicourses, as they unfolded.

The Conference portion consisted of about seven lectures for each of the three days, ranging from 45-55 minutes each. The lecturers included some of the most distinguished people in the field. Topics ranged over a broad spectrum, from theoretical physics to topology, geometry, algebra, etc., and included reports about some of the most recent important developments in the field such as the recent solution of the Poincaré Conjecture by Pereleman et al., to name just one example.

One somewhat unique feature that was greatly enjoyed by all was the almost daily concert of Indian Classical Music, following each session. Overall I would say the meeting was a great success. However it is always good to look for areas of possible improvement. One such, with an eye to the future of this important part of mathematics in India, would have been participation by a somewhat larger number of graduate students and young topologists. Another would have been a somewhat greater international participation. However, considering the time constraints from the inception of the idea to the meeting itself, the final result was about as good if not better than could have been expected and, as already mentioned, an excellent and successful meeting.

Respectfully submitted,
Peter Zvengrowski

Department of Mathematics and Statistics
University of Calgary
Calgary T2N 1N4 CANADA

## 2. Technical Report by P. Sankaran

Report on the International Workshop/Conference on Geometry and Topology, August 17th to 25th, 2007.

The Workshop was held from 17th to 22nd August. There were four speakers, namely, Professors K. Y. Kam, K. Varadarajan, P. Wong and P. Zvengrowski. Each of them gave five lectures, each lecture was of 90min duration.

Professor Lam lectured on 'Topological methods in the study of bilinear forms'. In these lectures he started with the work of Hurwitz and Radon concerning existence of bilinear maps of type (k,n,n) with norm property and led upto the frontiers of current research including his recent work on the topic.

Professor Varadarajan lectured on Wall's finiteness obstruction. Starting from basic notions of homotopy theory, algebraic K theory, ideal class groups, etc., he gave broad ideas of the theory as well as his contribution (jointly with Guido Mislin).

Professor Wong lectured on 'Combinatorial and geometric group theoretic methods in fixed point theory'. Starting with Nielsen fixed point theory, he moved on to Reidemeister classes and the relation with twisted conjugacy classes in groups. This is the meeting place of the various topics appearing in his title. His lectures led upto some recent result of his in the theme.

Professor Zvengrowski lectured on $l^2$ homology. Starting with basic results in the theory of bounded linear operators on a Hilbert space, Hilbert G-modules and von Neumann dimension, Kaplansky trace, he went on to define $l^2$ homology of a space and describe the axiomatics of this homology theory. The main application was to the theory of group presentation – particularly obtaining upper bounds deficiency for certain groups.

All the speakers made great efforts to reach out to the audience. They tried to relate to concepts in each others lectures. All the speakers made typed lecture notes available before the beginning of the workshop. The notes were bound and distributed to the audience at the time of registration (on day one).

Professor Shastri took care of the logistics and arranged several cultural programmes through out. This allowed the participants to feel relaxed despite the very hectic and packed schedule.

P. Sankaran,
Institute of Mathematical Sciences, Chennai.

## 3. Technical Report by P. Wong

This is my report on my participation in the International Workshop/Conference in Geometry and Topology during August 17th, 2007 at the Indian Institute of Technology Bombay, hosted by the Mathematics Department.

As one of the four workshop presenters, I was joined by Prof. K.Y. Lam of the University of British Columbia, Canada, Prof. K. Varadarajan and Prof. P. Zvengrowski, both of the University of Calgary, Canada. I gave 5 lectures on Combinatorial and Geometric Group Theoretic Methods in Fixed Point Theory.

In the first lecture, I gave a brief overview of topological fixed point theory, in particular, on Nielsen fixed point theory. The vanishing of the classical Lefschetz number of a map is not sufficient to deform the given map to be fixed point free. The more subtle invariant, namely the Nielsen number, often provides sufficiency. Lectures II and III discussed the Reidemeister trace and some of the existing combinatorial techniques (in the sense of combinatorial group theory) concerning the computation of the Reidemeister trace or the Nielsen number. Lectures IV and V focused on the property for groups. This property, which says that every automorphism has an infinite number of twisted conjugacy classes of elements, has its origin related to fixed point theory. If a finitely generated group $\pi$ has the property then one should not expect to obtain Jiang type results for self homeomorphism on a compact manifold with $\pi$ as the fundamental group. Combinatorial techniques involving commutator analysis were discussed and employed in constructing certain finitely generated torsion-free nilpotent groups with property. In the last lecture, geometric techniques regarding quasi-isometries were presented. We described the geometry of the lamplighter groups by means of their corresponding Deistel-Leader graphs.

Several techniques and concepts presented in other lectures by Lam, Varadarajan, and Zvengrowski during the workshop were also present in fixed point theory. For instances, cohomology with local coefficients, which is essential in the Wall's finiteness obstruction (KV lectures), is also essential in defining the primary obstruction to deforming a map to be fixed point free where the coefficients form a group ring with certain action of the fundamental group. Moreover, while nilpotent spaces are an integral part in the study of Wall's obstruction, certain nilpotent spaces and their generalization (C-nilpotent) have been shown to be Jiang-type spaces so that computation of the Nielsen number is feasible. Reidemeister trace also arises in the context of Bass' conjectures, one of which is equivalent to the equality between the classical Lefschetz number of a homotopy idempotent on a closed oriented manifold of dimension at least 3 and the - Lefschetz number of a lift to the universal cover. The - Lefschetz number is defined using the - (co)-homology which is the topic of Prof. Zvengrowski's lectures.

During the conference, I gave a lecture on generalizations of Fox homotopy groups and related groups of homotopy classes. One of the topics mentioned was the generalized Gottlieb group which was first introduced by Prof. K. Varadarajan in 1969.

In sum, lectures from both the workshop and from the conference represent a wide spectrum of topics in geometry and topology. It is worth noting that there were several recurrence, in terms of content and

techniques of some of the central themes among many of the lectures presented at the meeting.

Peter Wong
Bates College, Maine, USA.

## 4. Technical Report by K. Y. Lam

In preparing the lecture materials for the IITB workshop on geometry and topology, I had in mind an audience consisting of postgraduate students, young faculty members and researchers in relevant fields who are interested in seeing how techniques of algebraic topology can be applied to mathematical problems of general nature, such as equation solving. I decided to focus on the topic of nonsingular bilinear maps; it is related on the one hand to sums of squares, a topic in algebra of almost universal interest, and on the other hand to theorems of Borsuk-Ulam type which lie in the heartland of homology and homotopy theory. The mathematics department of IITB and the workshop coordinators rendered tremendous help by compiling my draft lecture notes into a volume (together with the notes of other workshop speakers ), and distributing it in advance to the audience. This single compilation makes it much more effective for lecturing. It saves plenty of time and allows the lecturer to focus on the essential subject matter. I must thank the organizing committee, and Prof. A. Shastri in particular, for their invaluable help in this regard.

While it seems to be the case that too much material had been cramped into the lecture notes, I believe my last minute decision to omit some sections in those notes had been an appropriate one. It allowed me to deliver the hard core sections in a way sufficiently intelligible, I hope, to the audience.

Participating in this workshop has been very rewarding for myself. Besides the opportunity to listen to the many workshop and conference speakers, I have had a first hand chance to witness the pride, optimism and dedication so evident in the young generation of Indian mathematicians, towards their academic pursuit and their cultural heritage. All told, I am thankful for the invitation to serve as workshop speaker, and I look forward to further exchanges with the many colleagues with whom I had made acquaintance during this visit to Bombay.

Kee Yuen Lam
University of British Columbia, Vancouver, Cananda.
September, 2007

# Unaudited Statement of Accounts

Please see the attachments for the detaled unaudited statement of accounts. The preparation for the programme started with the seed money of Rs. 50,000 granted by the Deputy Director of the Institute. The entire amount was returned as soon as the we received grants from DST (Rs. 2.5 lakh) and from NBHM (Rs. 1.5 lakh). Indeed DST has granted a some 3.2 lakhs. Since several participants could fund their travel on their own, and some likely participants could not make it, we are actually left with some access fund. The entire un-utilized fund will be returned to the respective agencies in due time along with the fully audited statment of accounts.