Seminar Meeting on Hyperbolic and Parabolic Partial Differential Equations

November 20 - 23rd, 2009

Titles and Abstracts

Date : 20/11/2009

Venue: Ramanujan Hall, Department of Mathematics

Session 1

9:30 AM to 11:00 AM

Chairman: Prof. G. D. Veerappa Gowda

9:30 AM to 10:15 AM

Speaker:

Prof. Tommaso Ruggeri
Faculty of Engineering, University of Bologna.
Currently, Visiting Professor in Department of Mathematics, IIT Bombay.
email: ruggeri@ciram.unibo.it

Title: Entropy Principle and Hyperbolic Dissipative Systems

Abstract: Recently, non-equilibrium theories and in particular extended thermodynamics have generated a new interest in quasi-linear hyperbolic system of balance laws with dissipation due to the presence of production terms (system with relaxation). On this subject it is very important to find connections between properties of the full system and the associated subsystem obtained when certain parameters (relaxation coefficients) are equal to zero. The requirement that the system of balance laws satisfies an entropy principle with a convex entropy density gives strong restrictions. Here we give a brief summary on these results with a particular attention to the local and global well-posedness of the relative Cauchy problem for smooth solutions and to the stability of constant solutions. Then we discuss the open question of Riemann problem for dissipative hyperbolic systems and we present a conjecture about the asymptotic behaviour for large time of the solution of the Riemann problem with and without structure. The conjecture is tested by numerical simulations.

10:15 AM to 11:00 AM

Speaker:
Prof. Phoolan Prasad
Department of Mathematics
Indian Institute of Science, Bangalore.
email: prasad@math.iisc.ernet.in

Title: Three-dimensional kinematical conservation laws - Part I: theory

Abstract: 3-D kinematical conservation laws (KCL) are equations of evolution of a moving surface Ω_t in 3-dimensional $(x_1, x_2, x_3)-$space $\mathbb{R}^3$. The KCL are derived in a specially defined ray coordinates (ξ_1, ξ_2, t), where ξ_1, ξ_2 are surface coordinates on Ω_t and $t > 0$ in time. Though the 3-D KCL were derived by Giles, Prasad and Ravindran in 1995, the theory of this system remained incomplete. Later a full analysis of the 3-D KCL system was completed by Arun and Prasad and published in 2008. Here we discuss various properties of 3-D KCL systems. KCL are the most general equations in conservation form, governing the evolution of t with singularities which we call kinks and which are curves on Ω_t in $\mathbb{R}^3$. Across a kink the normal $\boldsymbol{n}$ to Ω_t and the velocity m of Ω_t are discontinuous. From 3-D KCL we derive a system of six differential equations and show that the KCL system is equivalent to the ray equations for Ω_t. The six independent equations and an energy transport equation for small amplitude waves in a polytropic gas involving an amplitude w, which is related to the normal velocity m of Ω_t, form a completely determined system of seven equations. We have determined eigenvalues of the system by a novel method and find that the system has two distinct nonzero eigenvalues and five zero eigenvalues and the dimension of the eigenspace associated with the multiple eigenvalue zero is only four. For an appropriately defined m, the two nonzero eigenvalues are real when $m > 1$ and pure imaginary when $m < 1$.

Session 2

11:30 AM to 1:00 PM

Chairman: Prof. Jasobanta Jena

11:30 AM to 12:15 PM

Speaker:

Prof. Adimurthi
Tata Institute of Fundamental Research
Bangalore.
email: aditi@math.tifrbng.res.in

Title: Non existence of TV bounds for conservation laws with discontinuous fluxes

Abstract: For continuous scalar conservation laws, constant data gives constant solutions. In view of this, there exist TV bounds for a solution with data in BV. Here we show that this fails in the discontinuous fluxes along the curves of discontinuity.

12:15 PM to 1:00 PM

Speaker:

Prof. G. D. Veerappa Gowda
Tata Institute of Fundamental Research
Bangalore.
email: gowda@math.tifrbng.res.in

Title: Monotone Schemes for Conservation laws with flux function discontinuous in space variable.

Abstract: In this work we explain how to extend monotone schemes of conservation laws with continuous flux case to discontinuous flux case. In particular Godunov,Enquist-Osher and Lax-Friedrichs schemes. Convergence analysis of monotone schemes are studied for discontinuous flux case.

Session 3

2:30 PM to 4:00 PM

Chairman: Prof. Adimurthi

2:30 PM to 3:15 PM

Speaker:

Prof. Jasobanta Jena
Department of Mathematics
Netaji Subhas Institute of Technology
New Delhi.
email: istam09@gmail.com

Title: Lie Group of Transformations to Study Anisotropic Diffussion of Scalar Images

Abstract: Lie group of transformations associated with anisotropic diffusion equation is derived using the invariance properties. The equation is of great importance in the study of geometrical diffusion of scalar images. Infinitesimals of Lie group of transformations with respect to independent and dependent variables along with invariant surface conditions are used to obtain a general form of edge stopping function. The edge stopping functions used by Perona & Malik and their successors are obtained in particular cases. The equation in three independent variables reduced in terms of fewer independent variables.

3:15 PM to 4:00 PM

Speaker:

Prof. Ch. Radha
Department of Mathematics
University of Hyderabad
email: repakar@yahoo.com

Title: Similarity Solutions for Converging Shocks in a Relaxing Gas

Abstract: Using Lie group invariance method, a class of self-similar solutions are determined to a problem concerning plane and radially symmetric flows of a relaxing gas involving shocks of arbitrary strength. The ambient gas ahead of the shock is considered to be inhomogeneous. The method yields a general form of the relaxation rate for which

the self-similar solutions are admitted. The arbitrary constants, occurring in the expressions for the generators of the local Lie group of transformations, give rise to different cases of possible solutions with a power law, exponential or logarithmic shock paths. A particular case of the collapse of an imploding shock is worked out in detail for radially symmetric flows.

Session 4

4:30 PM to 5:15 PM

Chairman: Prof. Phoolan Prasad

4:30 PM to 5:15 PM

Speaker:

Dr. K. R. Arun
Department of Mathematics
Indian Institute of Science
Bangalore.
email: arunkr@math.iisc.ernet.in

Title: Three dimensional kinematical conservation laws-part II: application to high frequency nonlinear wave propagation
(*Jointly with Phoolan Prasad, IISc*)

Abstract: We study the propagation of a three-dimensional weakly nonlinear wavefront into a polytropic gas in a uniform state and at rest. The successive positions and geometry of the wavefront are obtained by solving the conservation form of equations of a weakly nonlinear ray theory. The proposed set of equations forms a weakly hyperbolic system of seven conservation laws with an additional vector constraint analogous to the scalar solenoidal condition in the equations of ideal magnetohydrodynamics. The new divergence free type constraint is termed as geometric solenoidal constraint. For the numerical simulation of the conservation laws we employ a high resolution central scheme. A constrained transport technique is used to enforce the geometric solenoidal constraint. The results of several numerical experiments are presented which confirm the efficiency and robustness of the proposed numerical method.

Date: 21/11/2009

Venue: Ramanujan Hall, Department of Mathematics

Session 1

9:30 AM to 11:00 AM

Chairman: Prof. Phoolan Prasad

9:30 AM to 10:15 AM

Speaker:

Prof. A. S. Vasudeva Murthy
Tata Institute of Fundamental Research
Bangalore.
email: vasu@math.tifrbng.res.in

Title: Numerical Solution for the Heat Equation in Unbounded domains.

Abstract: Numerical computation of solutions to differential equations on unbounded domains require the truncation of unbounded domain to a bounded domain. Boundary conditions imposed on the truncated domain are called artificial boundary conditions (ABCs). ABCs for the heat equation and its numerical analysis will be discussed.

10:15 AM to 11:00 AM

Speaker:

Prof. S. V. Raghurama Rao
CFD Centre
Department of Aerospace Engineering
Indian Institute of Science
Bangalore.
email: raghu@aero.iisc.ernet.in

Title: Exact Shock Capturing and Entropy Stable Central Schemes for Hyperbolic Conservation Laws

Abstract: Exact Shock Capturing Central Schemes developed recently (Journal of Computational Physics, vol. 228, pp. 770-798, 2009), as simpler alternatives to complicated flux splittings and Riemann solvers, are further improved by including a novel numerical entropy stability concept. The details of both these new developments are presented in this lecture, together with numerical simulations for hyperbolic systems of equations representing gas dynamics.

Session 2

11:30 AM to 1:00 PM

Chairman: Prof. A. S. Vasudeva Murthy

11:30 AM to 12:15 PM

Speaker:

Prof. J. C. Mandal
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Mumbai.
email: mandal@aero.iitb.ac.in

Title: Novel Upwind Method for Incompressible Flow Computations using Pseudo-Compressibility Approach

Abstract: Incompressible Navier-Stokes equations have a wide range of engineering applications like aerodynamics, hydrodynamics, internal flows, biomedical flows, motion associted with wind, etc. whose solutions have been of great interest to engineers and researchers. Numerous methodologies for obtaining the solution of these governing equations for different applications have been developed. Efforts are still on to develop better and accurate methods.

One of the most common technique to solve incompressible NS equations is the Artificial Compressibility method (also known as pseudo-compressibility method) initially introduced by Chorin[1] for steady state solutions. In the recent times, the pseudo-compressibility method has drawn considerable attention. All the advanced algorithm available for compressible flows are extendable to incompressible flows to obtain accurate solution in an efficient manner. Furthermore, this approach is also found to possess superior convergence[2].

Although, pseudo-compressibility method has been extensively used for last two decades, mostly two approaches, namely Jameson method and Roe method are used for the finite volume discretization of the inviscid fluxes. As Jameson finite volume method uses central diffferencing for inviscid fluxes, dissipative terms are added to stabilize the system. Since the dissipative terms are added externally, they need to be tuned for each problem depending on physics and geometry of the problem. On the other hand, the Roe's scheme is found to be quite accurate and does not depend on such tuning parameter. However, as the inviscid flux vector in pseudo-compressibility formulation is not

homogeneous function of degree one of the field vector, the Roe flux difference splitting is not proper. Although the results reported in the literature do not show the effect of the above deficiency in Roe's method so far, it may be worthwhile to look for an alternate formulation.

In this paper, we propose a novel Harten Lax and van Leer with contact (HLL-C)[3] type upwind method for pseudo-compressibility formulation of incompressible flow equations. Here, a three wave structure is considered for the formulation of the new upwind method[4-5]. Since, the eigenvalues in pseudo-compressibility formulation is subsonic like case of compressible flow, no discontinuous solutions are expected in the flow near steady state. As the solution approaches a steady state, the wave speed can be estimated very accurately unlike its compressible counterpart. Therefore, the new method is expected to be an accurate modelling for pseudo-compressibility formulation.

The proposed method is validated by a series of standard test cases including 3D flows with and without heat transfer[4-5]. All the computations are performed on unstructured grids in this paper. Although, complete quadrilateral elements are chosen for some test cases, data is handled in an unstructured fashion. The results obtained from the new method are very promising.

References:

1. Chorin A.J., A numerical method for solving incompressible viscous flow problems, Journal of Computational Physics, 1967, vol.2, pp.12-26.
2. Tamamidis P., Zhang P.G., and Assanis D.N., Comparison of pressure based and artificial compressibility methods for solving 3D steady incompressible viscous flows, Journal of Computational Physics, 1996, vol.124, pp.1-13.
3. Harten A., Lax P.D. and van Leer B., On Upstream Differencing and Godunov-Type Scheme for Hyperbolic Conservation Laws, SIAM Review, 1983, vol.25, No. 1, p.35.
4. Mandal J.C. And Iyer A., An upwind method for incompressible flow computations using pseudo-compressibility approach, AIAA Paper AIAA 2009-3541, 2009.
5. Mandal J.C. And Iyer A., An Upwind Method for Incompressible Flows With Heat Transfer, 1st Int. Conf. on Computational Methods for Thermal Problems, September 8-10, 2009, Naples, Italy.

12:15 PM to 1:00 PM

Speaker:
Prof. S. M. Deshpande
Engineering Mechanics Unit
Jawaharlal Nehru Centre for Advanced Scientific Research
Jakkur
Bangalore
email: smd@jncasr.ac.in

Title: Accuracy and Robustness of Kinetic Meshfree Method(KMM)
(*Joint work with Konark Arora1 CFD Division, DOCD, DRDL, Hyderabad - 500058 konark.arora@gmail.com)*

Abstract: Meshfree methods are gaining popularity over the conventional CFD methods for computation of inviscid and viscous compressible flows past complex configurations. The main reason for the growth of popularity of these methods is their ability to work on any point distribution. These methods donot require the grid for flow simulation, which is an essential requirement for all other conventional CFD methods. However these methods are limited by the requirement of a good connectivity around a node. Here, a very robust form of the meshfree method called Weighted Least Squares Kinetic Upwind Method using Eigendirections (WLSKUM- ED) has been used to avoid the problem of code divergence due to the bad connec- tivity. In WLSKUM-ED, the weights are calculated to diagonalize the least squares matrix A(w) such that the x and y directions become the eigen directions along which the higher dimensional least squares formulae reduce to the corresponding one dimensional formulae. Here an effort has been made to explain the enhanced robustness of the WLSKUM-ED meshfree method over the conventional LSKUM meshfree method. The accuracy of the kinetic meshfree method for the Euler equa- tions has been enhanced by use of entropy variables and inner iterations in the defect correction method. It is observed that the use of entropy variables and inner iterations in the defect correction method helps in obtaining the formal order of accuracy in case of a non-uniform point distribution.

Session 3

2:30 PM to 4:00 PM

Chairman: Prof. S. V. Raghurama Rao

2:30 PM to 3:15 PM

Speaker:
Prof. K. T. Joseph
Tata Institute of Fundamental Research
Mumbai.
email: ktj@math.tifr.res.in

Title: On δ- waves in cosmology

Abstract: We discuss initial value problems and initial boundary value problems for some systems of partial differential equations appearing in the modelling for the large scale structure formation in the universe. We restrict the initial data to be bounded measurable and locally BV function and use Volpert product to justify the product which appear in the equation. For more general initial data in the class of generalized functions of Colombeau, we construct the solution in the sense of association.

3:15 PM to 4:00 PM

Speaker:
Dr. Xiaoyu Fu
Department of Mathematics
Indian Institute of Technology Bombay
Mumbai.
email: xiaoyufu@math.iitb.ac.in

Title: A unified treatment for controllability and observability of PDEs

Abstract: In this report, from a basic identity of partial differential operator we deduce the known controllability and observability results for the parabolic, hyperbolic, Schrodinger and plate equations that are derived via Carleman estimate. In this sense, we give a unified treatment on controllability and observability problems for PDEs.

Session 4

4:30 PM to 5:15 PM

Chairman: Prof. V. D. Sharma

4:30 PM to 5:15 PM

Speaker:

Dr. T. Raja Sekhar
Department of Mathematics
Nation Institute of Technology
Rourkela. **email:** tungalar@nitrkl.ac.in

Title:

Abstract:

Date :23/11/2009

Venue: Ramanujan Hall, Department of Mathematics

Session 1

9:30 AM to 11:00 AM

Chairman: Prof. T. Ruggeri

9:30 AM to 10:15 AM

Speaker:

Prof. A. K. Pani
Department of Mathematics
Indian Institute of Technology Bombay
Mumbai.
email: pani@math.iitb.ac.in

Title: Oldroyd Model of Viscoelastic Fluids: Some Theoretical and Computational Issues

Abstract: Newton's model of incompressible viscous fluid is described by the wellknown Navier-Stokes equations. This has been a basic model for describing flow at moderate velocities of majority of viscous incompressible fluids encountered in practice. However, models of viscoelastic fluids have been proposed in the mid twentieth century which take into consideration the prehistory of the flow and are not subject to Newtonian flow. One such model was proposed by J.G. Oldroyd and hence, it is named after him. The equation of motion in this case gives rise to the following integro-differential equation

$$\frac{\partial u}{\partial t} + u \cdot \nabla u - \mu \Delta u - \int_0^t \beta(t-\tau) \Delta u(x,\tau)\, d\tau + \nabla p = f(x,t),\ x \in \Omega, t > 0, (*)$$

and incompressibility condition

$$\bigtriangledown . u = 0, \quad x \in \Omega, t > 0$$

with initial condition

$$u(x,0) = u_0, u = 0,\ x \in \partial\Omega, t \geq 0.$$

Here, Ω is a bounded domain in R^2 with boundary $\partial\Omega, \mu > 0$ and the kernel $\beta(t) = \gamma exp(-\delta t)$, where both γ and δ are positive constants. With a brief discussion on the model, we, in the first part of this talk, concentrate on a recently derived uniform estimates in time whose proof was bothering us for the last 10 to 12 years. With a brief

note on existence, we look into some new regularity results under realistically assumed conditions on the initial data. In the second part, we apply finite element Galerkin approximations to the above system and discuss convergence analysis without assuming compatibility conditions which are hard to verify while conducting numerical experiments. Since the problem (*) is an integral perturbation of the Navier-Stokes equations, we would like to discuss '*how far the results on finite element analysis for the Navier-Stokes equations can be carried over to the present case.*' We conclude the talk with some theoretical and computational issues.

10:15 AM to 11:00 AM

Speaker:
Prof. J. C. Mandal
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Mumbai.
email: mandal@aero.iitb.ac.in

Title: High Resolution Finite Volume Method Using Solution Dependent Weighted Least Squares (SDWLS) Gradient Estimation

Abstract: Development of high resolution schemes in order to capture discontinuities in the solutions of hyperbolic equations has been a challenge. Classical higher order schemes suffer from the problem of spurious oscillations near the discontinuities. For the last few decades, mainly two types of approaches have been employed in developing higher order schemes for hyperbolic equations. These approaches avoid spurious oscillations by bringing in non-linearities into the discretization process. In the first approach, known as total variational diminishing (TVD) method [1], the higher-order correction terms are added after multiplying them with limiter functions. The limiters, being a function of solution, help in adding the higher-order correction terms in a non-linear manner. In the second approach, known as essentially non-oscillatory (ENO) method [2], the non-linearity is brought in with the use of variable stencils, depending on least oscillatory criterion. Although these two methods yield quite satisfactory results for structured grids, their extension to multidimensions on unstructured grids is not very straightforward and perfect. Therefore, a necessity has been felt to look for an alternative approach. A novel approach, known as Solution Dependent Weighted Least Squares (SDWLS) method [3,4], has been introduced keeping primarily unstructured grid computations in mind. Here, the gradient terms in the reconstruction steps are approximated

with a weighted least-squares formulation, with the weights as functions of the solution. Thus, the non-linearity in the discretization is introduced through weighted least-squares estimation of gradient terms. The primary motivating factor for considering least-squares formulation in this approach is its ability to handle arbitrary sets of data points arising in unstructured grid-based flow computations. Initial attempts at implementation of higher order upwind schemes on unstructured grids focused on the extension of one-dimensional reconstruction procedure based on the MUSCL approach. Such an extension typically requires the use of one additional cell beyond the nearest neighbors. In the case of unstructured grids, consideration of additional neighbors to construct slopes for the higher-order part may introduce substantial complexity into the method and additional storage requirements. In unstructured grid computations, the computational stencil is thus usually restricted to the nearest neighbors. Keeping this in mind, the least-squares approach for the estimation of the gradients within each control volume has been used to obtain higher-order accuracy without informal inquiries beyond the nearest neighbors. The SDWLS technique, originally developed for second-order variable reconstruction [3], and further extended to third-order reconstruction using defect correction technique for one- and two-dimensional problems [4], by the present authors, has been modified further to improve the third order reconstruction procedure. Several standard two and three-dimensional test cases are computed with the SDWLS gradient estimation in order to demonstrate the efficacy of the present method. Significant improvemnets from second to third order reconstructions are observed in terms of accuracy and convergence in some cases.

References:

1. Harten A. High resolution schemes for hyperbolic conservation laws. Journal of Computational Physics 1983; 49:357393.

2. Harten A, Osher S, Engquist B, Chakravarthy SR. Some results on uniformly high-order accurate essentially non-oscillatory schemes. Applied Numerical Mathematics 1986; 2:347376.

3. Mandal JC, Subramanian J. On the link between weighted least-squares and limiters used in higher-order reconstructions for finite volume computations of hyperbolic equations. Applied Numerical Mathematics 2008; 58:705725.

4. J. C. Mandal, S. Rao and J. Subramanian. High-resolution finite volume computations using a novel weighted least-squares formulation. Int. J. Numer. Meth. Fluids 2008; 56:14251431.

Session 2

11:30 AM to 1:00 PM

Chairman: Dr. Xiaoyu Fu
11:30 AM to 12:15 PM

Speaker:
Prof. S. Baskar
Department of Mathematics
Indian Institute of Technology Bombay
Mumbai
email: baskar@math.iitb.ac.in

Title: Kinematical Conservation Laws and application to Sonic boom

Abstract: The structure of the sonic boom produced by a simple aerofoil at a large distance from its source consists of a leading shock (LS), a trailing shock (TS) and a one parameter family of nonlinear wavefronts in between the two shocks. We present a new formulation of the one parameter family of Cauchy problems to capture the nonlinear wavefronts using Weakly nonlinear ray theory (WNLRT) and the two shocks (LS and TS) using Shock ray theory (SRT). Our method takes in to account the nonlinearity from the very beginning. Also, the two theories are more robust and accurate than Whitham's Geometrical shock dynamics and hence gives a new direction of research in the field of sonic boom.

The propagation of a front from the aerofoil to the ground is governed by a systems of conservation laws (different systems for shock fronts and wavefronts derived from Kinematical conservation laws) which are hyperbolic for the fronts emerging from the leading part of the aerofoil and are elliptic for those from the trailing part. The transition from the hyperbolic to elliptic is a small region in which the wavefront is linear. This change in the nature of the system gives topologically different shapes to the fronts emerging from leading and trailing part of the aerofoil, which are also observed in the numerical solution of the Euler equations. In this work, we present in detail the formulation of the one parameter family of Cauchy problems and study the shape of the fronts numerically in the region where the governing system is hyperbolic. We make some conjecture about the shape of the fronts from the trailing part of the aerofoil based on our mathematical formulation of the problem with the numerical evidence of the solutions of Euler equations available in the literature.

12:15 PM to 1:00 PM

Speaker:

Prof. V. D. Sharma
Department of Mathematics
Indian Institute of Technology Bombay
Mumbai
email: vsharma@math.iitb.ac.in

Title:

Abstract:

Session 3

2:30 PM to 4:00 PM

Chairman: Prof. Ch. Radha

2:30 PM to 3:15 PM

Speaker:

Prof. T. Suman Kumar
Department of Mathematics
University of Hyderabad
email: suman.hcu@gmail.com

Title: Age Structured Population Dynamics

Abstract: The structured equations appear in many areas of population biology. The limitation of resources introduced by Verhulst leading to nonlinear integral forms. Age structured equations (renewal equations) seem the easiest to begin with.

First we obtain a priori bounds for competition term and use fixed point argument to get existence and uniqueness of renewal equations. The stability analysis leads to a spectral problem that can not be solved analytically in general. We give various structures showing that the steady state may be stable or unstable.

Finally we study the convergence of a numerical scheme for this model. Here the difficulty comes from the nonlinear birth term in the boundary condition and the absence of BV in the natural variable. This requires an alternate approach via BV estimates in time to deduce the compactness necessary to pass to the limit.

3:15 PM to 4:00 PM

Speaker:

Mr. Deepjyoti Goswami
Department of Mathematics
Indian Institute of Technology Bombay
Mumbai
email: deep@math.iitb.ac.in

Title: Optimal L2-error estimates for the semidiscrete Galerkin approximation to a second order linear parabolic initial and boundary value problem with nonsmooth initial data.

Abstract: In this presentation, we have discussed a priori error estimate for the semidiscrete Galerkin approximation of a general second order parabolic initial and boundary value problem with non-smooth data. Our analysis is based on energy argument without resorting to parabolic duality technique. This technique is also extended to semidiscrete mixed method for parabolic problems.

Session 4

4:30 PM to 5:15 PM

Chairman: Prof. A. K. Pani
4:30 PM to 5:15 PM

Speaker:
Ms. Sangita Yadav
Department of Mathematics
Indian Institute of Technology Bombay
Mumbai

Title: An *hp*-Local Discontinuous Galerkin Method for Parabolic Integro-Differential Equations
(*Joint work with A. K. Pani*)

Abstract: In this article, a priori error analysis is discussed for an *hp*-local discontinuous Galerkin (LDG) approximation to a parabolic integro-differential equation. It is shown that the L^2-norm of the gradient and the L^2-norm of the potential are optimal in the discretizing parameter h and suboptimal in the degree of polynomial p. Due to the presence of the integral term, an introduction of an expanded mixed type Ritz-Volterra projection helps to achieve optimal estimates. Further, it is observed that a negative norm estimate of the gradient plays a crucial role in our convergence analysis. As in the elliptic case, similar results on order of convergence are established for the semidiscrete method after suitably modifying the numerical fluxes. The optimality of these theoretical results is tested in a series of numerical experiments on two dimensional domains.