MA 401, Linear Algebra, Autumn 2007

Tutorial 1

1. Check whether the following subset of R^4 is linearly independent.

$$\{(1,1,0,0),(1,0,1,0),(0,0,1,1),(0,1,0,1)\}.$$

2. Let V be a vector space. If $\{v_1, v_2, v_3\} \subset V$ is linearly indepeendent, then show that $\{v - 1 + v_2, v_2 + v_3, v_3 + v_1\}$ is also linearly independent.

3. Let V be a vector space and $\{v_1, v_2, v_3\} \subset V$. Show that

$$\text{span}\ \{v_1, v_2, v_3\} = \text{span}\ \{v_1 + v_2, v_2, v_3\}.$$

4. Let V be the vector space of all $n \times n$ matrices. Check whether the following sets form a subspace of V.
   (a) All invertible matrices.
   (b) All non-inverible matrices.
   (c) All matrices A such that $A^2 = A$.
   (d) Let B be a fixed matrix in V and $S = \{A \in V : AB = BA\}$.

5. Find bases of the following vector spaces.
   (a) The vector space of all real $n \times n$ upper triangular matrices.
   (b) The vector space of all real $n \times n$ symmetric matrices.
   (c) The vector space of all real $n \times n$ skew-symmetric matrices.
   (d) The vector space $\{(x_1, x_2, \cdots, x_n) \in R^n : x_1 + x_2 + \cdots + x_n = 0\}$.

6. Let V be the vector space of 2×2 matrices. Find a basis $\{A_1, A_2, A_3, A_4\}$ for V such that $A_i^2 = A_i,\ i = 1, 2, 3, 4$.