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On the SL(2) **period integral. (English summary)**
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The study of period integrals of automorphic forms is an important subject in the Langlands program. More precisely, given a cuspidal representation π on a reductive l-group G containing a subgroup H, one says that π is H-distinguished if the period integral over H defines a nonzero linear functional on π. When E is a quadratic extension of a number field F and $(G,H)=(\mathrm{GL}_n(E),\mathrm{GL}_n(F))$, this question has been much studied. In this case, a cuspidal representation π is H-distinguished iff π is conjugate self-dual and its Asai L-function has a pole at $s=1$. We should mention that there is an analogous local question whose study often illuminates the global question above. In the local setting, one would like to know the dimension of the space of H-invariant linear functionals on π. If, for example, this dimension is at most 1 for all places, then the global period integral admits a factorization into the product of the local functionals.

In the paper under review, the authors examine the question of distinguishedness for $(G,H)=(\mathrm{SL}_2(E),\mathrm{SL}_2(F))$. The situation is quite delicate because of the nontriviality of L-packets for SL_2, which reflects the fact that a representation of GL_2 does not in general remain irreducible when restricted to SL_2. The authors' main results are:

(i) If a cuspidal representation π of $\mathrm{SL}_2(E)$ is contained in the restriction of a $\mathrm{GL}_2(F)$-distinguished cuspidal representation of $\mathrm{GL}_2(E)$, then π is distinguished with respect to $\mathrm{SL}_2(F)$ iff it has a ψ-Whittaker coefficient for an automorphic character ψ of A_E trivial on A_F (here A_E is the adèle ring of E).

(ii) There are cuspidal representations of $\mathrm{SL}_2(E)$ which are locally distinguished at every place but not globally distinguished. Indeed, there are cuspidal representations of $\mathrm{SL}_2(E)$ which are locally distinguished everywhere, but for which no members in its global L-packet are globally distinguished.

(iii) The global period integral over $\mathrm{SL}_2(F)$ does not in general admit a factorization into the product of local functionals. A precise criterion for when it does factor is given.

The proofs of these results depend on a delicate study of the restriction of automorphic representations from GL_2 to SL_2 (and also some natural intermediate subgroups) and do not make further use of powerful machinery such as trace formulae or functorial lifting, etc. They also rely on an earlier paper of the authors in which they resolved the analogous local problem [see Math. Res. Lett. **10** (2003), no. 5-6, 867–878; MR2025061].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.