Let $F$ be a $p$-adic field and let $G$ be the group of $F$-rational points of a connected quasi-split reductive $F$-group. Let $P = MN$ be a parabolic subgroup. Let $\psi : W'_F \to L M < L G$ be an Arthur parameter of $G$ which factors through $L M$, where $W'_F$ is the Deligne-Weil group and $L G$ (resp. $L M$) denotes the Langlands dual of $G$ (resp. $M$). Thus $\psi$ is an Arthur parameter for $M$. Assume that the image of $\psi$ is not contained in a smaller Levi subgroup. Let $S_{\psi}$ be the centralizer in $\tilde{G}$ of the image of $\psi$, and let $S^o_{\psi}$ denote its identity component. Let $T_{\psi}$ be a maximal torus of $S^o_{\psi}$. Define $W_{\psi} = N_{S_{\psi}}(T_{\psi})/Z_{S_{\psi}}(T_{\psi})$ and $W^o_{\psi} = N_{S^o_{\psi}}(T_{\psi})/Z_{S^o_{\psi}}(T_{\psi})$. Let $\sigma$ be an irreducible unitary representation of $M$ which belongs to the $A$-packet $\Pi_{\psi}(M)$. Let $W(\sigma) = \{ w \in W(G,A) \mid w \sigma \cong \sigma \}$, where $A$ is the split component of $M$. Let $W_{\psi,\sigma} = W_{\psi} \cap W(\sigma)$ and $W^o_{\psi,\sigma} = W^o_{\psi} \cap W(\sigma)$. The Arthur $R$-group is defined as $R_{\psi,\sigma} = W_{\psi,\sigma}/W^o_{\psi,\sigma}$. Note that Arthur’s characterization of the $R$-group does not require $\sigma$ to be in the discrete series whereas the classical Knapp-Stein $R$-group requires such a constraint as it is defined in terms of Plancherel measures. In a number of situations, the Arthur $R$-group is known to match the Knapp-Stein $R$-group if $\sigma$ is in the discrete series.

Let $I = \text{Ind}_{S^o_{\psi}}^G \sigma$ be the parabolically induced representation of $G$. Let $A'(\sigma, \tilde{w})$ be Shahidi’s normalized intertwining operator, where $\tilde{w}$ is a representative of $w$. If $w \in W(\sigma)$, $\sigma$ extends to a representation $\sigma_w$ of the smallest group containing $M$ and $\tilde{w}$. Define $A(\sigma, w) = \sigma_w(\tilde{w}) A'(\sigma, \tilde{w})$. The definition is independent of the choices involved and this defines an element in the commuting algebra $C(\sigma) = \text{Hom}(I, I)$. From the cocycle condition satisfied by Shahidi’s operator, it follows that $A(\sigma, w_1 w_2) = \eta(w_1, w_2) A(\sigma, w_1) A(\sigma, w_2)$ for a constant $\eta(w_1, w_2)$.

Now let $R$ be a subgroup of $W(\sigma)$. There is a finite central extension $1 \to Z_{\alpha} \to \tilde{R} \to R \to 1$ which splits $\eta$. Let $\xi : \tilde{R} \to \mathbb{C}^\times$ be the function which splits $\eta$. There is a homomorphism from $\tilde{R}$ to $C(\sigma)$ and to each component $\pi$ of $I$, there is an attached representation $\rho_{\pi}$ of $\tilde{R}$. For $r \in R$, define $\langle r, \pi \rangle = \text{trace} \rho_{\pi}(r, 1)$. Then the authors prove that

$$\text{trace}(A(\sigma, r) I(f)) = \xi(r) \sum_{\pi} \langle r, \pi \rangle \text{trace} \rho_{\pi}(f)$$

where $f \in C_c^\infty(G)$. For $\sigma$ in the discrete series, this is a well-known property of the Knapp-Stein $R$-group.

The authors also show that two other basic properties of the Knapp-Stein $R$-group do not carry over to the Arthur $R$-group. Thus, the normalized standard intertwining operators $A(r, \sigma), r \in R_{\psi,\sigma}$ in general do not form a basis for the commuting alge-
bra and the components of $I$ are not in bijective correspondence with the irreducible representations of $R_{\psi,\sigma}$. This is shown by considering $\pi = \text{Ind}_P^G(\text{St}_{\text{GL}(2)} \otimes 1_{\text{GL}(2)})$, where $G = \text{SO}(9), P = MN$ with $M = \text{GL}(2) \times \text{GL}(2)$. It is shown that $\pi$ has three components whereas $R_{\psi,\sigma} = \mathbb{Z}/2 \times \mathbb{Z}/2$.

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\textit{Classification :}

\begin{itemize}
  \item 11F70 Representation-theoretic methods in automorphic theory
  \item 22E50 Repres. of Lie and linear algebraic groups over local fields
\end{itemize}