This paper surveys some aspects of the theory of periods of automorphic forms on a reductive group. A period of an automorphic form on $G$ is essentially its integral over a subgroup $H$. Automorphic representations admitting a nonvanishing period integral, the so-called distinguished representations, are of considerable interest. For instance, nonvanishing of the period integral is related to functoriality in certain contexts and in some situations it is related to special $L$-values. The local analogue of this notion is expressed in terms of the existence of a nontrivial invariant linear form on the space of the local representation. The global condition ensures that the local criterion is met at all the places. One can ask when will, and under what extra assumptions, the local period condition guarantees the global nonvanishing. The paper discusses all these points with a lot of examples.

The periods of much interest in this paper are the so-called generalized Gelfand-Graev models, which are based on the theory of nilpotent orbits. Let $O$ be a nilpotent orbit in $\mathfrak{g} = \text{Lie}(G)$. Then there is a standard parabolic $P_O = M_O N_O$ such that the Lie algebra $\mathfrak{n}_O$ is a graded $\mathfrak{m}_O$-module $\mathfrak{n}_1 \oplus \mathfrak{n}_2 \oplus \cdots$ and the intersection of $O$ with $\mathfrak{n}_2$ is an open dense $\mathfrak{m}_O$-orbit. Let $M_O^\circ$ be the subgroup of $G$ corresponding to the stabilizer of this open orbit. For the sake of simplicity, let us assume that $\mathfrak{n}_1 = 0$. If $\varphi$ is an automorphic form on $G$, the Fourier coefficient of $\varphi$ attached to $O$ is defined by $\mathcal{F}_{\psi O}(g, \varphi) = \int \varphi(ng) \psi^{-1}_O(n) \, dn$, where the integral is over $N_O$. Then $\mathcal{F}_{\psi O}$ defines an automorphic form on $M_O^\circ$. Now let $H$ be a subgroup of $M_O^\circ$ and let $\phi$ be an automorphic form on $H$. Then the Gelfand-Graev period integral in this situation is given by $\int_H \mathcal{F}_{\psi O}(h, \varphi) \phi(h) \, dh$. In this context, the author surveys his work with D. Ginzburg and S. Rallis on orthogonal groups [J. Am. Math. Soc. 17, No. 3, 679–722 (2004; Zbl 1057.11029); Ohio State Univ. Math. Res. Inst. Publ. 11, 157–191 (2005; Zbl 1115.11027)]. In this case, nonvanishing of the above period integral is related to the central value of the Rankin-Selberg $L$-function being nonzero. This is also related to the Gross-Prasad conjecture.

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