

Q1. To fill a tank, 25 buckets of water is required. How many buckets of water will be required to fill the same tank if the capacity of the bucket is reduced to two-fifth of its present ?

- (A) 10
- (B) 35
- (C) 62.5
- (D) cannot be determined

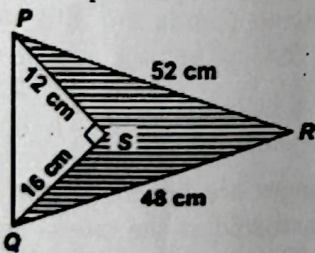
Q2. A man covered a certain distance at some speed. Had he moved 3 km/hr faster, he would have taken 40 minutes less. If he had moved 2 km/hr slower, he would have taken 40 minutes more. The distance (in km) is:

- (A) 35 km
- (B) $110/3$ km
- (C) $75/2$ km
- (D) 40 km

Q3. A man has some hens and cows. If the number of heads be 48 and the number of feet equals 140, then the number of hens will be:

- (A) 22
- (B) 23
- (C) 24
- (D) 26

Q4. In the adjoining figure, the area of shaded portion is:



- (A) 98 cm^2
- (B) 480 cm^2
- (C) 384 cm^2
- (D) 380 cm^2

Q5.
$$\frac{(469+174)^2 - (469-174)^2}{(469 \times 174)} =$$

- (A) 2
- (B) 4
- (C) 295
- (D) 643

Q6. Which of the following statements is correct ?

- (A) If $x^6 + 1$ is divided by $x + 1$, then the remainder is -2 .
- (B) If $x^6 + 1$ is divided by $x - 1$, then the remainder is 2.
- (C) If $x^6 + 1$ is divided by $x + 1$, then the remainder is 1.
- (D) If $x^6 + 1$ is divided by $x - 1$, then the remainder is -1 .

Q7. In a mixture 60 litres, the ratio of milk and water is 2 : 1. If this ratio is to be 1 : 2, then the quantity of water to be further added is:

- (A) 20 litres
- (B) 30 litres
- (C) 40 litres
- (D) 60 litres

Q8. The ratio of the number of girls & boys in a class is 8 : 7. If the % age increase in the number of girls & boys be 10% & 20% respectively, what will be the new ratio?

- (A) 21 : 22
- (B) 22 : 21
- (C) 1 : 1
- (D) Can not be determined

Q9. If the circumference of a circle and the perimeter of a square are equal, then

- (A) Area of circle = Area of square
- (B) Area of circle < Area of square
- (C) Area of circle > Area of square
- (D) Insufficient information

Q10. Rahul was asked his age by his friend. Rahul said "The number you get when you subtract 25 times my age from twice the square of my age will be thrice your age. If the friend's age is 14, then the age of Rahul is

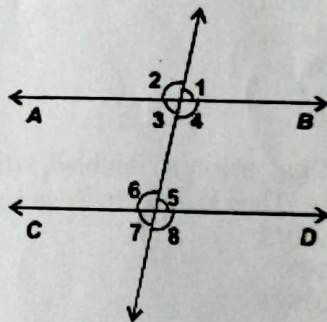
- (A) 20
- (B) 22
- (C) 18
- (D) 14

Q11. Pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13 are:

- (A) 58 and 13 or 16 and 29
- (B) 68 and 23 or 36 and 49
- (C) 18 and 73 or 56 and 93
- (D) 78 and 13 or 26 and 39

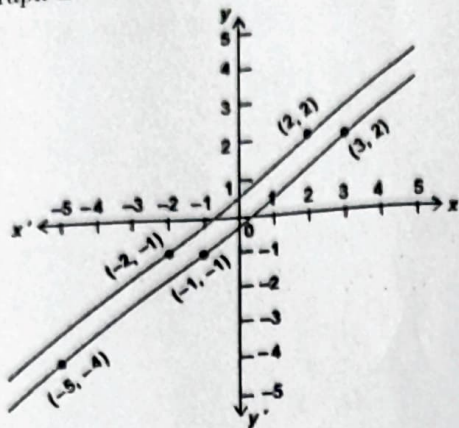
- Q12. Seven times of a two digit number is equal to four times the number obtained by reversing the order of digits and the sum of the digits is 3. The number is:
 (A) 12
 (B) 10
 (C) 15
 (D) 20
- Q13. Given three lines
 (i) $L_1 : a_1x + b_1y + c_1 = 0$
 (ii) $L_2 : a_2x + b_2y + c_2 = 0$
 (iii) $L_3 : (a_1 + a_2)x + (b_1 + b_2)y + (c_1 + c_2) = 0$
 If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then line L_3 is
 (A) Parallel to line L_1
 (B) Parallel to line L_2
 (C) is coincident with L_1 or L_2
 (D) none of these
- Q14. A towel, when bleached, was found to have lost 20% of its length and 10% of its breadth. The percentage of decrease in area is:
 (A) 10%
 (B) 10.08%
 (C) 20%
 (D) 28%
- Q15. Simplify : $\sqrt[5]{\sqrt[4]{(2^4)^3}} - 5\sqrt[5]{8} + 2\sqrt[5]{\sqrt[4]{(2^3)^4}}$
 (A) $-2\sqrt[5]{(2)^3}$
 (B) $\sqrt[5]{(2)^3}$
 (C) $2\sqrt[5]{(2)^3}$
 (D) $-\sqrt[5]{(2)^3}$
- Q16. The last digit of 3^{400} is :
 (A) 1
 (B) 3
 (C) 2
 (D) 0
- Q17. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?
 (A) Rs 460
 (B) Rs 484
 (C) Rs 550
 (D) Rs 664
- Q18. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?
 (A) $\frac{1}{10}$
 (B) $\frac{2}{5}$
 (C) $\frac{2}{7}$
 (D) $\frac{5}{7}$
- Q19. A new club flag is to be designed with 6 vertical stripes using some or all of the colours yellow, green, blue and red. In how many ways can this be done so that no two adjacent stripes have the same colour.
 (A) $2 \cdot 3^5$
 (B) $4 \cdot 3^5$
 (C) $5 \cdot 3^4$
 (D) $4 \cdot 3^4$
- Q20. Two numbers are respectively 20% and 50% more than a third number. The ratio of the first two numbers is :
 (A) 2 : 5
 (B) 3 : 5
 (C) 4 : 5
 (D) 6 : 7
- Q21. The sum of three numbers is 98. If the ratio of first to second is 2 : 3 and that of the second and third is 5 : 8, then the second number is:
 (A) 20
 (B) 30
 (C) 48
 (D) 58
- Q22. 100 oranges are bought at the rate of Rs350 and sold at the rate of Rs48 per dozen. The percentage of profit or loss is:
 (A) 100/7% gain
 (B) 15% gain
 (C) 100/7% loss
 (D) 15% loss
- Q23. A trader mixes 26 kg of rice at Rs. 20 per kg with 30 kg of rice of other variety at Rs. 36 per kg and sells the mixture at Rs. 30 per kg. His profit percent is:
 (A) No profit, no loss
 (B) 5%
 (C) 8% loss

- (D) 10%
- Q24. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is:
 (A) 50 km
 (B) 56 km
 (C) 70 km
 (D) 80 km
- Q25. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 km away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is:
 (A) 100 km/hr
 (B) 110 km/hr
 (C) 120 km/hr
 (D) 130 km/hr
- Q26. A boy multiplied 12345679 by second, third and seventh multiple of 9, then the average of their total is:
 (A) 44444444
 (B) 4444444
 (C) 4444444
 (D) 444444
- Q27. In how many years will a sum of Rs. 800 at 10% per annum compounded semi-annually become Rs. 926.10 ?
 (A) $1\frac{1}{3}$
 (B) $1\frac{1}{2}$
 (C) $2\frac{1}{3}$
 (D) $2\frac{1}{2}$
- Q28. Probability of an event can be:
 (A) -0.7
 (B) $\frac{11}{9}$
 (C) 1.00001
 (D) 0.9999
- Q29. If the radius of a circle is decreased by 50%, the percentage decrease in its area is:
 (A) 60%
 (B) 75%
 (C) 70%
 (D) 80%
- Q30. A hollow garden roller of height 63cm, with a girth of 440cm is made of iron 4cm thick. The volume of the iron used is
 (A) 54982 cm^3
 (B) 56372 cm^3
 (C) 57636 cm^3
 (D) 107712 cm^3
- Q31. Find the remainder when $9x^3 - 3x^2 + x - 5$ is divided by $x - \frac{2}{3}$.
 (A) 3
 (B) -3
 (C) 2
 (D) -2
- Q32. If $4^{44} + 4^{44} + 4^{44} + 4^{44} = 4^x$, then x is:
 (A) 45
 (B) 44
 (C) 11
 (D) 176
- Q33. In the given figure, $AB \parallel CD$. If $\angle 1 = (2x + y)^\circ$ and $\angle 6 = (3x - 7)^\circ$, then the measure of $\angle 2$ in terms of y is:



- (A) $(108 - y)^\circ$
 (B) $(2 - y)^\circ$
 (C) $(1 - y)^\circ$
 (D) $(100 + y)^\circ$

Q34. The equations representing the given graph is:



- (A) $7x + 2y = 11$; $y - 2x = 3$
 (B) $2x + 7y = 11$; $4x + 17.5y = 25$
 (C) $3x - 7y = 10$; $8y - 6x = 4$
 (D) $3x - 4y = 1$; $8y - 6x = 4$

Q35. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

- (A) $10/21$
 (B) $11/21$
 (C) $2/7$
 (D) $5/7$

Q36. If selling price is doubled, the profit triples. Then the profit %age is:

- (A) $200/3$
 (B) 100
 (C) $316/3$
 (D) 120

Q37. In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is:

- (A) 5 km/hr
 (B) 6 km/hr
 (C) 6.25 km/hr
 (D) 7.5 km/hr

Q38. A man has Rs. 480 in the denominations of one-rupee notes, five-rupee notes and ten-rupee notes. The number of notes of each denomination is equal. What is the total number of notes that he has?

- (A) 45
 (B) 60
 (C) 75
 (D) 90

Q39. If $a - b = 3$, and $a^2 + b^2 = 29$, then the value of ab is:

- (A) 10
 (B) 12
 (C) 15
 (D) 18

Q40. A sum of Rs1360 has been divided among A, B and C such that A gets $2/3$ of what B gets and B gets $1/4$ of what C gets. B's share is:

- (A) Rs 120
 (B) Rs 160
 (C) Rs 240
 (D) Rs 300

Level 1

1) Suppose $S = \{m \in \mathbb{Z}: m = a^2 + b^2 \text{ for some } a, b \in \mathbb{Z}\}$, i.e., M denotes collection of integers which are the sum of two perfect squares.

- A. if $p, q \in S \Rightarrow pq \in S$.
- B. if $p > q \in S \Rightarrow p - q \in S$.
- C. Both 1, 2 are correct.
- D. Both 1, 2 are incorrect.

2) Suppose $a, b, c \in \mathbb{R}$ such that

$$a + b + c = 0$$

$$a^3 + b^3 + c^3 = 0$$

Then which of the following is correct?

- A. Then $a^2 + b^2 + c^2 = 0$.
- B. There exist infinitely many solutions such that $abc \neq 0$.
- C. There exists a unique solution such that $abc \neq 0$
- D. $abc = 0$.

3) Suppose

$$A = 2^{22222} + 22222^2$$

$$B = 22^{2222} + 2222^{22}$$

$$C = 222^{222} + 222^{222}$$

Then which of the following is correct?

- A. $B < C < A$
- B. $C < B < A$
- C. $C < A < B$.
- D. $A < C < B$

4) $a_i \in \mathbb{R} - \{-1\}$ and $a_i \neq a_j$ for $i \neq j$ where $i, j = 1, 2, \dots, n$.

$$\frac{1}{1+a_1} + \frac{2}{1+a_2} + \dots + \frac{n}{1+a_n} = 95, \text{ and}$$

$$\frac{a_1}{1+a_1} + \frac{2a_2}{1+a_2} + \dots + \frac{na_n}{1+a_n} = 115, \text{ Then find } n.$$

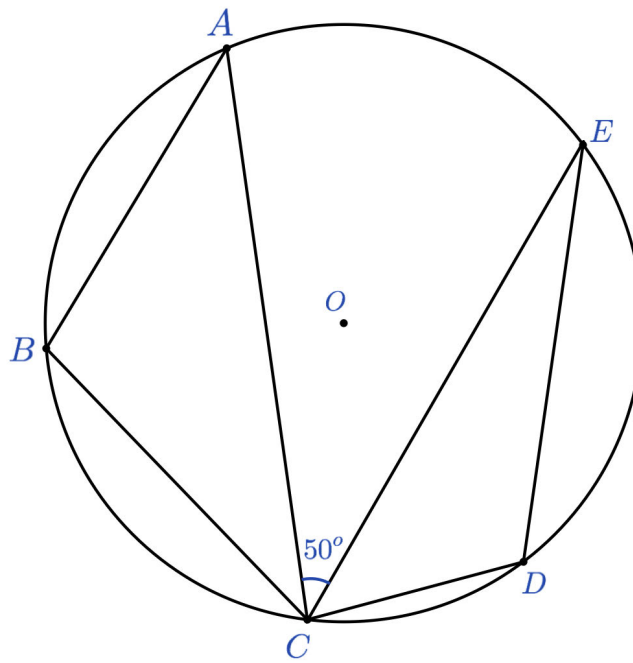
- A. 15
- B. 20
- C. 25
- D. 30

5) $2^x + 3^y = 11$ and $2^{x+2} + 3^{y+1} = 42$, then $xy = ?$

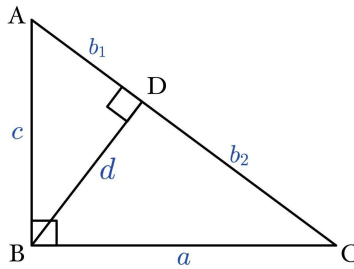
- A. 1
- B. 2
- C. 3
- D. 4

6) Consider a circle with center O . Suppose ABC and CDE are two triangles, all of whose vertices lie on the boundary of the circle. Assume $\angle ACE = 50^\circ$. Then what is the value of $\angle ABC + \angle CDE$

- A. 200°
- B. 230°
- C. 310°
- D. Cannot be concluded since the given information is insufficient.



7) In $\triangle ABC$, the angle B is of 90 degree. Let D be a point on line segment AC such that BD is an altitude (from B onto AC). Suppose $AB = c$, $BC = a$, $AD = b_1$, $CD = b_2$ and $BD = d$. Then which of the following is true?



- A. If $a = 20$, $c = 15$, then $b_1 = 9$, $b_2 = 16$ and $d = 12$.
- B. If $b_1 = 9$, $b_2 = 16$, then $a = 20$, $c = 15$ and $d = 12$.
- C. Both A and B
- D. None of Above

8) In a class of 15 boys and 10 girls, a test of 100 marks is conducted in which the average marks of the boys are found to be 78 and the class average is 82. After a crib session, three boys increased their marks to 6, 9, and 15 whereas two girls increased their marks to 4 and 6. Then the new average of the marks of the boys, girls and the class average is _____, _____ and _____, respectively.

- A. 80, 89 and 83.6
- B. 79, 89 and 83
- C. 79, 90 and 83.4
- D. 80, 90 and 84

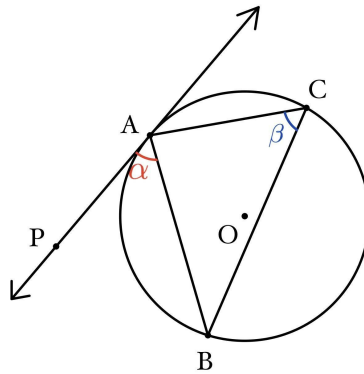
9) If five people are working together on one project then they complete it in 8 hours, and if four out of them are working together then they complete it in 10 hours. Then the fifth person alone can complete the same project in how many hours?

- A. 24 hours
- B. 30 hours
- C. 36 hours
- D. 40 hours

10) The set of real triples $\{(a, b, c) \mid a^2 + b^2 + abc^3 = 1\}$ is

- A. An empty set.
- B. Infinite set which is unbounded.
- C. A finite non-empty set.
- D. An infinite set which is bounded.

11) Let AB be a proper chord (that is, it is not diameter) of a circle with center O and PA be a tangent to the circle at a point A such that $\angle PAB$ is acute. Let C be a point of circle such that ACB is a major arc. Suppose $\angle PAB = \alpha$ and $\angle ACB = \beta$, then _____.



- A. $\alpha = \beta$
- B. $\alpha = 2\beta$
- C. $2\alpha = \beta$
- D. There is no relation between α and β

12) The last digit of $2022^{27} + 27^{2022}$ is _____.

- A. 1
- B. 3
- C. 7
- D. 9

13) A, B, and C can complete a work in 28, 20, and 35 days respectively. In how many days they can complete this work if they work together in the way that A will work only on $(3n+1)$ -th days, B will work only on $(3n+2)$ -th days, and C will work only on $(3n+3)$ -th days, where n varies over non-negative integers?

- A. 28
- B. 29
- C. 25
- D. 26

14) Suppose $AABABCB$ is a 7 digit positive natural number which is divisible by 3, 7 and 13. Then which of the following is correct?

- A. There exists a unique such positive natural number.
- B. There exists exactly 2 such positive natural numbers.
- C. There exists exactly 3 such positive natural numbers
- D. There exists no such positive natural numbers.

15) Suppose a function $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ satisfies

$$f(x) - f\left(-\frac{1}{x}\right) + f(-x) = x + \frac{1}{x}$$

Then which of the following is correct?

- A. $f(10) = 10.1$
- B. $f(10) = -10.1$
- C. $f(-10) = 11$
- D. $f(-10) = -11$

16) Consider the equations given below.

$$\sqrt[3]{10-x} + \sqrt[3]{30-x} = \sqrt[3]{25-x} + \sqrt[3]{15-x}$$

Then over \mathbb{R} , the equation has

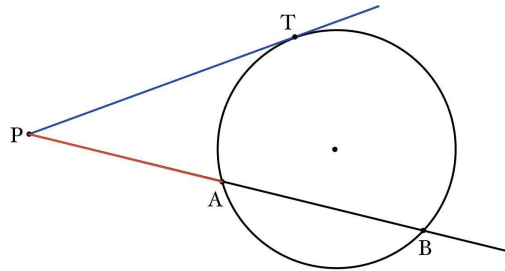
- A. No Solution
- B. Has a unique solution
- C. Has 2 distinct solutions
- D. Has 3 distinct solutions

17) If p is a prime number greater than 3, the equation $p^2 - 1 = 3n^3 + 12$ has

- A. No integer solution for n
- B. Infinitely many integer solutions for n
- C. Finitely many but non-empty integer solutions for n .
- D. None of the above

18) In a circle, let PT be a tangent to the circle, and PA be a secant line intersecting the circle at a points A and B . Then $2PT$ _____ $(PA) + (PB)$.

- A. =
- B. \leq
- C. \geq
- D. None of the above.



Level 2

1) Suppose a polynomial $p(x)$ leaves remainder n when divided by $x - n$ for $n = 1, 2, 3$ and 4 .

Then which of the following is correct?

- A. There exists a unique such polynomial of degree 3.
- B. There exists a unique such polynomial of degree 4.
- C. There exists no such polynomial.
- D. None of the above.

2) Consider the complex numbers z such that $|z| \leq 1/2$. Find the least possible value of $|z + 3 + 4i|$ correct up to 2 decimal places.

- A. 4.50
- B. 4.60
- C. 5.00
- D. 4.71

3) The series $1 + 1/\sqrt{3} + 1/\sqrt{5} + 1/\sqrt{7} + \dots$

- A. Converges to a number strictly between 4 and 5.
- B. Converges to a number strictly between 5 and 6.
- C. Converges to a number strictly between 6 and 7.
- D. Does not converge to any limit.

4) Let $x, y \in \mathbb{R}$. Consider the 3×3 matrix $A = (a_{ij})$ where (a_{ij}) is equal to x if $i+j$ is a multiple of 3 and is equal to y if $i+j$ is a multiple of 4 and is equal to 3 otherwise. Let $f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x, y) = \det(A)$. Then which of the following is correct?

- A. $g_1(y) = f(-3, y)$ has local minima at an integer point.
- B. $g_2(y) = f(-1, y)$ has local minima at an integer point.
- C. $g_3(y) = f(3, y)$ has local minima at an integer point.
- D. $g_4(y) = f(1, y)$ has local minima at an integer point.

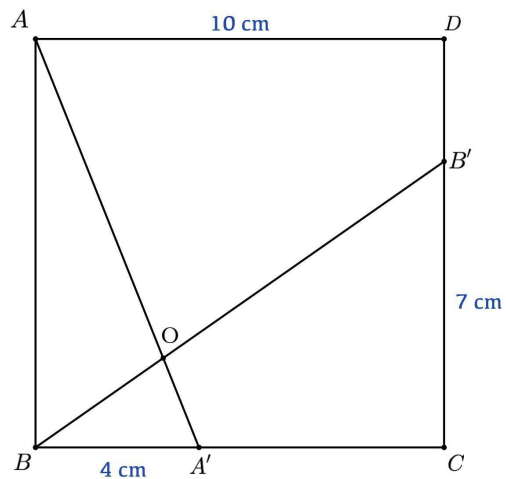
5) Let A be a set with cardinality 100. The number of subsets of A with odd cardinality is equal to _____.

- A. 2^{49}
- B. 2^{50}
- C. 2^{99}
- D. 2^{100}

6) The last digit of the number $1! + 2! + \dots + 2022!$ is _____.

- A. 1
- B. 3
- C. 5
- D. 7

7) Consider a square $ABCD$ of side 10 cm. Suppose that points A' and B' divide BC and CD , respectively as given in the diagram below. Suppose that AA' and BB' intersect at O . What is the



area of Triangle OBA' ?

- A. 5.375 cm^2
- B. 4.275 cm^2
- C. 4.375 cm^2
- D. 5.275 cm^2 .

8) If a 5-digit number has at least one digit 4, what is the probability that it is a palindrome number?

A palindrome number is a number (such as 16461) that remains the same when its digits are reversed.

A. $109/1000$

B. $2725/11878$

C. $5939/12500$

D. $7/1042$

9) For a natural number n , let $P(n)$ denote the probability that a positive integer less than n is a perfect cube. Then:

A. $P(n)$ tends to 1 as n tends to infinity.

B. $P(n)$ does not approach any limit as n tends to infinity.

C. $P(n)$ tends to 0 as n tends to infinity.

D. $P(n)$ tends to $\frac{1}{\sqrt{3}}$ as n tends to infinity.

10) For two non-zero complex number z and w , if $|z-w| = |z|-|w|$, then

A. z and w must be real numbers

B. w must be very small compared to z .

C. At least one of z and w is real.

D. $\frac{w}{z}$ is a real number.

11) For which interval of the values of m , all the roots of the quadratic equation $x^2 - (2m + 1)x + m^2 + 3m - 6 = 0$ are less than 3?

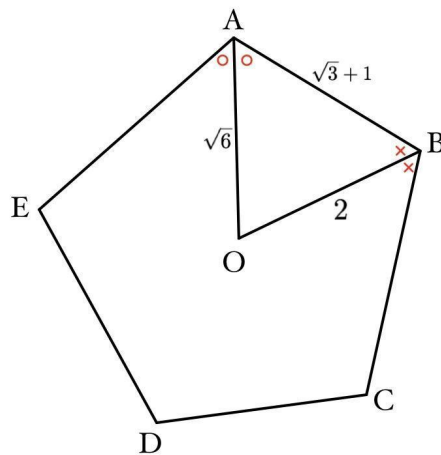
- A. $(-\infty, 2)$
- B. $(-6, 2)$
- C. $(-3, \infty)$
- D. $(-\infty, 0)$

12) Let α and β be positive real numbers less than 1. Then the sequence

$$\alpha^\beta, (2\alpha)^\beta, (3\alpha)^\beta, \dots$$

- A. Has no limit
- B. Has a limit strictly less than 1 which depends on α and β
- C. Always converges to 1
- D. Converges to a real number more than 1

13) Consider a convex pentagon $ABCDE$. The angle bisectors of A and B meet at an interior point O . Suppose length of $AB = \sqrt{3} + 1$, $AO = \sqrt{6}$ and $BO = 2$. Then the sum of angles C , D and E is _____.



- A. 300
- B. 330
- C. 360
- D. 390

14) Let A and B be two sets with cardinality 6 and 3, respectively. What is the probability that a given function from A to B is onto?

- A. 11/27
- B. 13/27
- C. 16/27
- D. 20/27

15) Suppose for every $\alpha \in \mathbb{R} \setminus \{0\}$, we are given a polynomial

$P_\alpha(x) = x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0$ ($\alpha_i \in \mathbb{R}$, $i = 0, 1, 2, 3$) of degree 4 with the property that α , $-\alpha$, $1/\alpha$ and $-1/\alpha$ are the roots of it. For $i \in \{0, 1, 2, 3\}$, consider the functions $F_i: \{P_\alpha \mid \alpha \in \mathbb{R} \setminus \{0\}\} \rightarrow \mathbb{R} \setminus \{0\}$, defined by $F_i(P_\alpha) = \alpha_i$. Then which of the following is true?

- A. Range of F_0 is infinite.
- B. Range of F_1 is infinite.
- C. Range of F_2 is infinite.
- D. Range of F_3 is infinite.

16) Consider the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = n/2$ if n is even, $f(n) = 3n + 1$ if n is odd. Note that function f^s means s -times composition of f (e.g. $f^3(n) = f(f(f(n)))$). A number n is said to be *good* if there exists some $k \in \mathbb{N}$ for which $f^k(n) = 2$. Then

- (i) There are infinitely many good odd numbers.
- (ii) There are infinitely many good even numbers.

(iii) There exists a natural number n_0 such that $f^k(n_0) = 8$ for infinitely many values of k .

(iv) There exists a natural number n_0 such that $f^k(n_0) = 9$ for infinitely many values of k .

Which of the following is true?

A. Only (i), (ii) and (iii) are correct.

B. Only (iii) and (iv) are correct.

C. Only (ii) is correct.

D. Only (i) and (ii) are correct.

Answer: D

17) Suppose 8 people A, B, C, D, E, F, G and H are sitting inside a cafe on a round table, and are drunk. Later they had a vague memory of the person they were sitting with. They put their points below and are trying to figure out their relative positions.

- 1) A : I was sitting with G .
- 2) B : I was sitting with E .
- 3) C : I was sitting with neither D nor F .
- 4) D : I wasn't sitting with G .
- 5) E : Well, I remember nothing
- 6) F : Even I don't remember anything
- 7) G : I was not sitting with C .
- 8) H : I was sitting with E

On the basis of their claims, which of the following conclusions are correct?

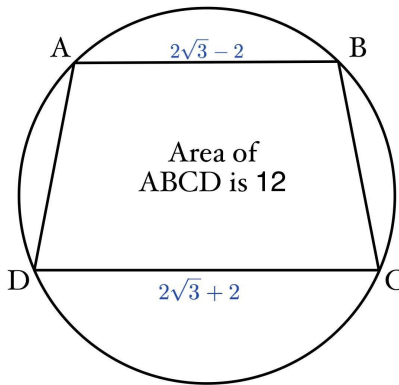
- A. F was sitting with B or H .
- B. D was sitting with A .
- C. D was sitting with B or H .
- D. F was sitting with A .

18) Let H be a finite set of distinct positive integers none of which has a prime factor greater than 3. Then the sum of the reciprocals of the elements of

H is

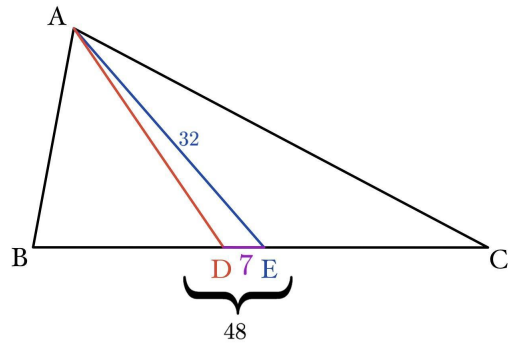
- A. Necessarily < 2
- B. Necessarily < 3 but can be more than 2
- C. Necessarily < 4 but can be more than 3
- D. Unbounded

19) Let $ABCD$ be a cyclic quadrilateral which is also a trapezoid such that the lines AB and CD are parallel. Given that the length of $AB = 2\sqrt{3} - 2$, $CD = 2\sqrt{3} + 2$ and the area of quadrilateral is 12. Then the area of the circle (passing through A, B, C, D) is _____.



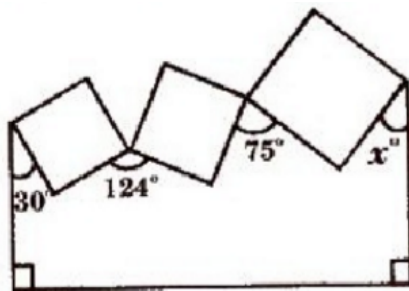
- A. 5π
- B. 6π
- C. 7π
- D. 8π

20) Given a triangle $\triangle ABC$, let AD be an angle bisector (of A) and AE be a median (from A to BC) such that $B - D - E - C$ is the position of points on the line segment BC . Suppose the length of $BC = 48$, $AE = 32$, $DE = 7$. Then what is $2AB + AC$?

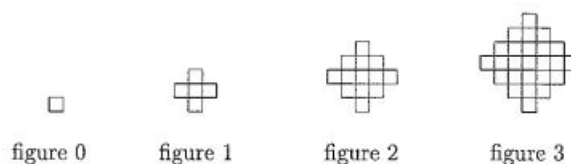


- A. 102
- B. 104
- C. 106
- D. 108

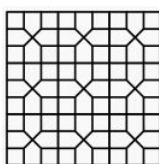
1. What is the ratio of the radii of the incircle (circle inscribed) to the circumcircle of the triangle whose sides measure 5, 12 and 13 units?
(a) $\frac{4}{13}$ (b) $\frac{6}{13}$ (c) $\frac{1}{2}$ (d) $\frac{12}{13}$
2. The Juhu beach has an average of 510 visitors on Sundays and 240 on other days. The average number of visitors per day in a month of 30 days beginning with a Sunday is
(a) 250 (b) 280 (c) 276 (d) 285
3. Tintisa takes a loan of Rs. 600 at 5% simple interest. She returns Rs. 200 at the end of 1st year, another 200 at the end 2nd year. In order to clear her dues at the end of 3rd year, she would pay
(a) 260 (b) 200 (c) 690 (d) 290
4. The weights of 3 heaps of sand are in the ratio 5 : 6 : 7. By what fraction of themselves should the first two heaps be increased respectively so that the ratio of the weights changes to 7 : 6 : 5.
(a) $\frac{12}{13}, \frac{24}{25}$
(b) $\frac{24}{25}, \frac{19}{20}$
(c) $\frac{2}{5}, \frac{19}{20}$
(d) $\frac{24}{25}, \frac{2}{5}$
5. Complete the sequence 6, 24, 60, 120, . . .
(a) 200 (b) 210 (c) 240 (d) 250
6. How many times will the hands of a clock coincide in a day?
(a) 24 (b) 22 (c) 20 (d) 21
7. If 75% of a class of 40 can sing, and only 20% cannot dance, the maximum number of students who can either not sing or not dance is:
(a) 0 (b) 2
(c) 18 (d) Cannot be determined
8. Suppose there are four socks in a drawer such that each of them could be either black or white. You take out two of the socks from the drawer. If the probability of drawing two black socks is $\frac{1}{2}$. What is the probability of drawing a pair of white socks from the drawer?
(a) 0 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) Cannot be determined
9. Three squares are joined at their corners and strung between two vertical poles as shown. What is the value of x ?



- (a) 52° (b) 36° (c) 30° (d) 41°
10. How many line segments have both their endpoints located at the vertices of a given cube?
- (a) 24 (b) 28 (c) 12 (d) 16
11. A subset B of the set of first 100 positive integers has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B ?
- (a) 63 (b) 64 (c) 62 (d) 61
12. Let a, b and c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. What is the value of $a^2 - b^2 + c^2$?
- (a) 2 (b) 5 (c) 7 (d) 1
13. Figures 0, 1, 2 and 3 consist of 1, 5, 13 and 25 nonoverlapping unit squares, respectively. If the pattern were continued, how many nonoverlapping unit squares would there be in figure 100?

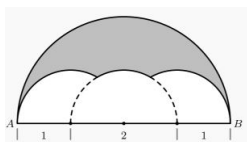


- (a) 10410 (b) 19801 (c) 20201 (d) 40801
14. How many distinct triangles with perimeter 7 have integer side lengths ?
- (a) 3 (b) 5 (c) 1 (d) 2
15. The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to

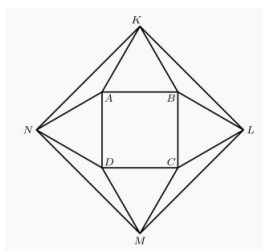


- (a) 50 (b) 58 (c) 52 (d) 56

16. The positive integers $A, B, A - B$, and $A + B$ are all prime numbers. The sum of these four primes is
 (a) even (b) divisible by 3 (c) divisible by 5 (d) prime
17. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?
 (a) 8 (b) 10 (c) 12 (d) 16
18. An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius?
 (a) 2 : 1 (b) 3 : 1 (c) 4 : 1 (d) 16 : 3
19. Three semicircles of radius 1 are constructed on diameter \overline{AB} of a semicircle of radius 2. The centers of the small semicircles divide \overline{AB} into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?

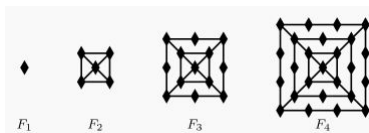


- (a) $\pi - \sqrt{3}$ (b) $\pi - \sqrt{2}$ (c) $\frac{\pi + \sqrt{3}}{2}$ (d) $\frac{7}{6}\pi - \frac{\sqrt{3}}{2}$
20. Points K, L, M and N lie in the plane of the square $ABCD$ such that AKB, BLC, CMD and DNA are equilateral triangles. If $ABCD$ has an area of 16, find the area of $KLMN$.

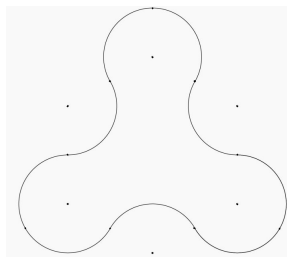


- (a) 32 (b) $16 + 16\sqrt{3}$ (c) 48 (d) $32 + 16\sqrt{3}$
21. Let f be a function with the following properties:
 (i) $f(1) = 1$, and
 (ii) $f(2n) = n \times f(n)$,
 for any positive integer n . What is the value of $f(2^{2017})$?
 (a) 1 (b) 2^{2016} (c) 2^{2017} (d) 2^{2017^2}
22. What is the area enclosed by the graph of $|3x| + |4y| = 12$?
 (a) 6 (b) 12 (c) 16 (d) 24

23. How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?
 (a) 41 (b) 43 (c) 44 (d) 45
24. Call a number prime-looking if it is composite but not divisible by 2, 3 or 5. The three smallest prime-looking numbers are 49, 77 and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?
 (a) 100 (b) 102 (c) 104 (d) 108
25. The figures F_1 , F_2 , F_3 and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20} ?



- (a) 401 (b) 485 (c) 626 (d) 761
26. Let n be the smallest positive integer such that n is divisible by 20, n^2 is a perfect cube, and n^3 is a perfect square. What is the number of digits of n ?
 (a) 3 (b) 4 (c) 7 (d) 6
27. Let x and y be two-digit positive integers with mean 60. What is the maximum value of the ratio $\frac{x}{y}$?
 (a) 3 (b) $\frac{33}{7}$ (c) $\frac{39}{7}$ (d) $\frac{99}{10}$
28. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012 a Tuesday. On what day of the week was Dickens born?
 (a) Friday (b) Saturday (c) Monday (d) Tuesday
29. The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



- (a) $2\pi + 6$ (b) $2\pi + 4\sqrt{3}$ (c) $2\pi + 3\sqrt{3} + 2$ (d) $\pi + 6\sqrt{3}$

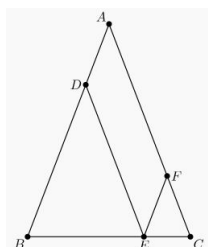
30. When counting from 3 to 2017, 53 is the 51st number counted. When counting backwards from 2017 to 3, 53 is the n^{th} number counted. What is n ?

- (a) 1967 (b) 1966 (c) 1965 (d) 1964

31. Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\dots$, with a and b different digits. What is the sum of the elements of S ?

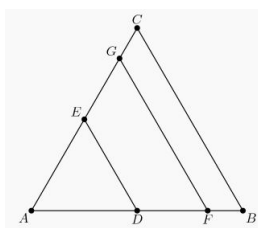
- (a) 11 (b) 44 (c) 143 (d) 155

32. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D , E , and F are on sides \overline{AB} , \overline{BC} , and \overline{AC} , respectively, such that \overline{DE} and \overline{EF} are parallel to \overline{AC} and \overline{AB} , respectively. What is the perimeter of parallelogram $ADEF$?



- (a) 48 (b) 56 (c) 60 (d) 72

33. Triangle ABC is equilateral with $AB = 1$. Points E and G are on \overline{AC} and points D and F are on \overline{AB} such that both \overline{DE} and \overline{FG} are parallel to \overline{BC} . Furthermore, triangle ADE and trapezoids $DFGE$ and $FBCG$ all have the same perimeter. What is $DE + FG$?



- (a) 1 (b) $\frac{3}{2}$ (c) $\frac{21}{13}$ (d) $\frac{13}{8}$

34. Which of the following divides $361^2 + 169^2 + 361 \cdot 169$?

- (a) 787 (b) 283 (c) 313 (d) non of the above

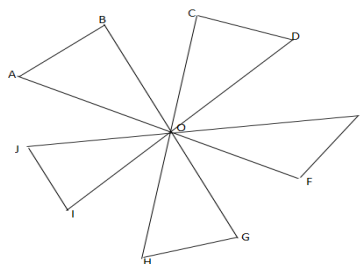
35. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

- (a) 35 (b) 75 (c) 105 (d) 25

36. Which of the following is true?

- (a) $\frac{22}{7}$ is a rational number and π is an irrational number

- (b) $\frac{22}{7}$ is an irrational number and π is a rational number
 (c) $\frac{22}{7}$ and π both are irrational number
 (d) $\frac{22}{7}$ and π both are rational number
37. Protik purchased sugar worth Rs 400. He sold $\frac{3}{4}$ th at a loss of 10% and the remainder at a gain of 10%. What is the net profit or loss ?
 (a) a loss of 5% (b) a gain of 5% (c) a loss of $\frac{96}{19}$ (d) a loss of $\frac{100}{19}$
38. Amit and Dibyendu can do a piece of work in 5 days, Dibyendu and Kalyan can do it in 7 days, Amit and Kalyan can do it in 4 days. Who among these will take the least time if put to do it alone?
 (a) Amit (b) Dibyendu (c) Kalyan (d) Data inadequate
39. A train travelling at 48 kmph completely crosses another train having half its length and travelling in opposite direction at 42 kmph, in 12 seconds. It also passes a railway platform in 45 seconds. The length of the platform is.
 (a) 400 m (b) 450 m (c) 560 m (d) 600 m
40. From the figure given below find out the value of
 $\angle OAB + \angle OBA + \angle OCD + \angle ODC + \angle OEF + \angle OFE + \angle OGH + \angle OHG + \angle OIJ + \angle OJI$



- (a) 640° (b) 540° (c) 360° (d) 720°

Bonus Question

41. A sports competition has n events and $2n$ participants. The first event eliminates half the participants, the second eliminates half of the remaining ones (a quarter of the total), and so on, until only an overall winner is left. Suppose a ranking of expected performance is made for each event separately, prior to the start of the tournament. A contestant is called a potential winner if, for some ordering of the events and assuming the expected ranking holds true, that contestant will win the tournament.
- (a) It is possible for 2^{n-1} contestants to be potential winners.
 (b) It is not possible for more than $2^n - n$ contestants to be potential winners.
 (c) It is possible for exactly $2^n - n$ contestants to be potential winners.
 (d) None

Space For Rough Work

Space For Rough Work

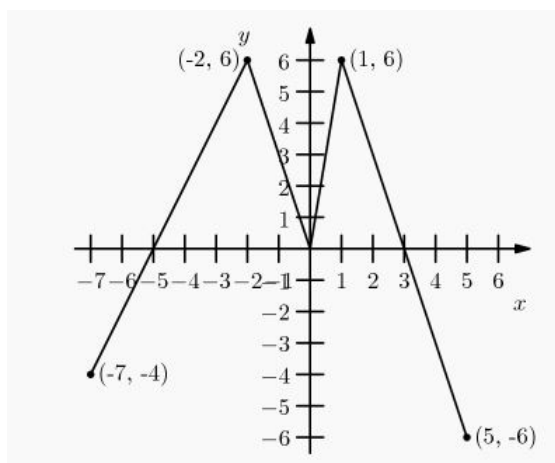
Notations

- \mathbb{N} denotes the set of natural numbers $\{1, 2, 3, \dots\}$, \mathbb{Z} - the integers, \mathbb{Q} - the rationals, \mathbb{R} - the reals and \mathbb{C} - the field of complex numbers. \mathbb{R}^n denotes the n -dimensional Euclidean space.
- The symbol $\mathbb{Z}/n\mathbb{Z}$ (or $\frac{\mathbb{Z}}{n\mathbb{Z}}$) will denote the ring of integers modulo n .
- The symbol S_n will denote the symmetric group on a finite set of n symbols.
- $n! = n \times (n - 1) \times (n - 2) \times \dots \times 2 \times 1$ and $0! = 1$.
- ${}^n P_r$ denotes the number of permutations of n objects taken r at a time.
- The symbol $\binom{n}{r}$ will denote the standard binomial coefficient giving the number of ways of choosing r objects from a collection of n objects, where $n \geq 1$ and $0 \leq r \leq n$ are integers.
- The symbol (a, b) will stand for the open interval $\{x \in \mathbb{R} \mid a < x < b\}$ while $[a, b]$ will stand for the close interval $\{x \in \mathbb{R} \mid a \leq x \leq b\}$; $[a, b)$ and $(a, b]$ will stand for the corresponding left-closed-right-open and left-open-right-closed intervals respectively.
- We denote by $M_n(\mathbb{R})$ (respectively, $M_n(\mathbb{C})$), the set of all $n \times n$ matrices with entries from \mathbb{R} (respectively, \mathbb{C}).
- The transpose of a matrix A will be denoted by A^t (or A^T), the trace of a square matrix A by $\text{tr}(A)$ and the determinant of a square matrix A by $\det(A)$.
- The derivative of a function f will be denoted by f' and the second derivative by f'' .

Part - A

(Only one option is correct)

- The dimensions of a floor are 18×24 . What is the smallest number of identical square tiles that will pave the entire floor without the need to break any tile ?
 (a) 6 (b) 24 (c) 12 (d) 8
- The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 6$ have?



- (a) 6 (b) 5 (c) 4 (d) 2
- The value of
$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 5^2 & 6^2 & 7^2 & 8^2 \\ 9^2 & 10^2 & 11^2 & 12^2 \\ 13^2 & 14^2 & 15^2 & 16^2 \end{vmatrix}$$

 (a) 0 (b) 1 (c) 2 (d) 3
- $\lim_{(x \rightarrow 0)} \sqrt{x^3 - x^2}$
 (a) 0 (b) 1 (c) -1 (d) Does not exist
- Determine the set of positive values of x that satisfy the following inequality

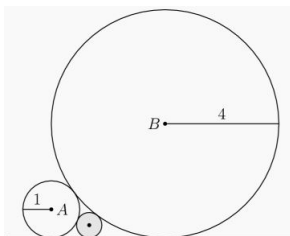
$$\frac{1}{x} - \frac{1}{x-1} > \frac{1}{x-2}$$

- (a) $(0, \frac{1}{2}) \cup (1, 2)$ (b) $(1, \sqrt{2}) \cup (\sqrt{2}, 2)$
 (c) $(1, \sqrt{2}) \cup (2, 2\sqrt{2})$ (d) $(0, 1) \cup (\sqrt{2}, 2)$
- A function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \begin{cases} 2x, & \text{if } x \in \mathbb{Q}, \\ 1-x, & \text{if } x \notin \mathbb{Q}. \end{cases}$
 Then what is the point of continuity?
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 0 (d) 1

7. $\int_{-1}^1 [x^2 - x + 1] dx$ equals: ($[x]$ denotes the greatest integer less than or equal to x)
- (a) $\frac{1-\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{5+\sqrt{5}}{2}$ (d) $\frac{5-\sqrt{5}}{2}$
8. The number of solutions of $\sin^{-1}(2x) = 3 \sin^{-1} x$ is
- (a) 0 (b) 1 (c) 2 (d) 3
9. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable at $x = a$ and let $f'(a) > 0$. Then the value of

$$\lim_{x \rightarrow a} \left(\frac{f(x)}{f(a)} \right)^{\frac{1}{\log x - \log a}}$$

- (a) $e^{a \frac{f'(a)}{f(a)}}$ (b) $e^{\frac{f'(a)}{f(a)}}$ (c) $e^{a \frac{f(a)}{f'(a)}}$ (d) $e^a \cdot e^{\frac{f'(a)}{f(a)}}$
10. A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. The radius of the third circle is



- (a) $\frac{5}{12}$ (b) $\frac{4}{9}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$
11. An isosceles triangle has an angle 120° and incircle radius $\sqrt{3}$ unit; then area of the triangle is
- (a) $7 + 12\sqrt{3}$ (b) $12 - 7\sqrt{3}$ (c) $12 + 7\sqrt{3}$ (d) 4π
12. What is the area of a quadrilateral in the xy -plane whose vertices are $(0,0), (1,0), (2,3)$ and $(0,1)$?
- (a) 2.5 (b) 3 (c) 3.5 (d) 5
13. The value of
- $$\sum_{n=1}^{\infty} \tan^{-1} \left(\frac{2n}{n^4 + n^2 + 2} \right)$$
- is
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) 1 (d) 0
14. If $z^2 + z + 1 = 0$, where z is a complex number; then the value of $(z + \frac{1}{z})^2 + (z^2 + \frac{1}{z^2})^2 + (z^3 + \frac{1}{z^3})^2 + \dots + (z^6 + \frac{1}{z^6})^2$ is:
- (a) 6 (b) 18 (c) 12 (d) 54
15. Let $E(n)$ denote the sum of the even digits of n . For example, $E(1243) = 2 + 4 = 6$. What is the value of $E(1) + E(2) + E(3) + \dots + E(100)$?
- (a) 402 (b) 400 (c) 398 (d) 404

16. What is the probability getting two aces in successions(with replacement) from a deck of 52 cards?
 (a) $\frac{1}{52}$ (b) $\frac{2}{169}$ (c) $\frac{2}{159}$ (d) $\frac{1}{169}$
17. *Statement I:* Let a, b, c be positive integers such that a divides b^4 , b divides c^4 and c divides a^4 . Then abc divides $(a + b + c)^{21}$.
Statement II: Let a, b, c be positive integers such that a divides b^3 , b divides c^3 and c divides a^3 . Then abc divides $(a + b + c)^{13}$.
 Which of the following option is true ?
 (a) (I) (b) (II) (c) Both (d) None
18. Let S be the set of all points with coordinates (x, y, z) , where x, y , and z are each chosen from the set $\{0, 1, 2\}$. How many equilateral triangles all have their vertices in S ?
 (a) 72 (b) 76 (c) 80 (d) 84
19. How many positive integers less than 1000 are 6 times the sum of their digits?
 (a) 0 (b) 1 (c) 4 (d) 12
20. A palindrome between 1000 and 10,000 is chosen at random. What is the probability that it is divisible by 7?
 (a) $\frac{1}{10}$ (b) $\frac{1}{7}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

Part - B

(One or more option can be correct)

21. If

$$\phi(\alpha, \beta) = \begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ \cos(\alpha + \beta) & -\sin(\alpha + \beta) & 1 \end{vmatrix}$$

then

- (a) $\phi(300, 200) = \phi(400, 200)$ (b) $\phi(200, 400) = \phi(200, 600)$
 (c) $\phi(100, 200) = \phi(200, 200)$ (d) None
22. Which of the following is/are real number(s)? ($i = \sqrt{-1}$)
 (a) $\left(\frac{1+i\sqrt{3}}{2}\right)^{2017}$ (b) $\left(\frac{1+i}{\sqrt{2}}\right)^{2017}$ (c) $(-1)^{(2017)}$ (d) $e^{(2017)i}$
23. For how many integers n , is $\frac{n}{20-n}$ the square of an integer?
 (a) finitely many (b) infinitely many (c) 4 (d) 6
24. For 3×3 matrices M and N which of the following statements is/are NOT correct?
 (a) $MN - NM$ is skew-symmetric for all symmetric matrices M and N .

- (b) $N^T MN$ is symmetric or skew-symmetric according as M is symmetric or skew-symmetric.
- (c) $\text{Adj}(M) \cdot \text{Adj}(N) = \text{Adj}(MN)$ for all invertible matrices M and N .
- (d) There exist two invertible matrices M and N such that $MN - NM = I$, where I is the 3×3 identity matrix.
25. Let $f(x) = x^2 + 6x + 1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x) + f(y) \leq 0 \quad \text{and} \quad f(x) - f(y) \leq 0$$

The area of R is closest to

- (a) 22 (b) 23 (c) 24 (d) 25
26. *Statement(I)*: Between any two rational number, there is an irrational number.
Statement(II): Between any two irrational number, there is a rational number .
- (a) Both (I) and (II) are true (b) (II) is true but (I) is false
 (c) (I) is true but (II) is false (d) Neither (I) or (II) are true
27. The equation $x^4 - 2ax^2 + x + a^2 - a = 0$ have all real solutions for which values of a ?
- (a) $(0, \infty)$ (b) $[\frac{3}{4}, \infty)$ (c) $(\frac{3}{4}, \infty)$ (d) $(0, \frac{3}{4})$
28. Let $f : [a, b] \rightarrow \mathbb{R}$ be any continuously differentiable function on $[a, b]$ and twice differentiable on (a, b) such that $f(a) = f'(a) = 0$ and f vanishes at b . Then
- (a) $f''(a) = 0$ (b) $f'(x) = 0$ for some $x \in (a, b)$
 (c) $f''(x) = 0$ for some $x \in (a, b)$ (d) $f'''(x) = 0$ for some $x \in (a, b)$
29. Consider the polynomial

$$P(x) = \prod_{k=0}^{10} (x^{2^k} + 2^k) = (x + 1)(x^2 + 2)(x^4 + 4) \cdots (x^{1024} + 1024)$$

The coefficient of x^{2012} is equal to 2^a . What is a ?

- (a) 5 (b) 6 (c) 7 (d) 10
30. Consider the two statements
Statement I : $\frac{\sin x}{x} < \sqrt{\cos x}$ for $x \in (0, \frac{\pi}{2})$
Statement II : $\sin^2 \theta < \theta \sin(\sin \theta)$ for $\theta \in (0, \frac{\pi}{2})$
- (a) (I) is true (b) (II) is true
 (c) Both (I) and (II) are true (d) Both (I) and (II) are false
31. Let, $f(x) = \begin{cases} \frac{\sin(x^2)}{x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0. \end{cases}$ Then

- (a) f is continuous for any value of $k \in \mathbb{R}$

- (b) f is continuous for the value of $k = 0$
 (c) $f'(0) = 0$ when $k = 0$
 (d) $f'(0) = 1$ when $k = 0$

32. Which of the following is/are true?

- (a) If A is a 3×3 square matrix with $\det A = 4$, then $\text{adj}(\text{adj} A) = 16A$
 (b) If A is a 3×3 square matrix with $\det A = 3$, then $\det(\text{adj}(\text{adj} A)) = 81$
 (c) If A is a 3×3 square matrix, $\text{adj}(3A) = k \cdot \text{adj}(A)$, then $k = 27$
 (d) If

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix},$$

then $\det(\text{adj} A) = 512$

33. Let $A(3, 5)$, $B(2, 2)$, $C(3, 1)$ be the vertices of a triangle $\triangle ABC$. Let O be the orthocentre of $\triangle ABC$. Then co-ordinates of the orthocentre of $\triangle BOC$ will be

- (a) $(\frac{3}{2}, \frac{1}{2})$ (b) $(\sqrt{2}, \sqrt{3})$ (c) $(0, 2)$ (d) None of these

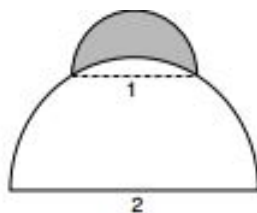
34. In an LPP, if the objective function is parallel to a constraint and the feasible region is non-empty, then the objective function may assume

- (a) an unbounded solution
 (b) unique optimal solution
 (c) multiple optimal solution
 (d) any one of the above

35. Which of the following is/are true?

- (a) The last digit of 7^{2017} is 7
 (b) The last two digits of 7^{2017} is 07
 (c) The last three digits of 7^{2017} is 607
 (d) None of the above

36. A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of the lune.



- (a) $\frac{\sqrt{3}}{4} + \frac{\pi}{12}$ (b) $\frac{\sqrt{3}}{4} - \frac{\pi}{12}$ (c) $\frac{\sqrt{3}}{4} - \frac{\pi}{24}$ (d) $\frac{\sqrt{3}}{4} + \frac{\pi}{24}$

37. A number n is chosen at random from $S = \{1, 2, 3, \dots, 50\}$. Let $A = \{n \in S : n + \frac{50}{n} > 27\}$, $B = \{n \in S : n \text{ is a prime}\}$ and $C = \{n : n \text{ is a square}\}$. The correct order of their probabilities is

- (a) $P(A) < P(B) < P(C)$ (b) $P(A) > P(B) > P(C)$
(c) $P(B) < P(A) < P(C)$ (d) $P(A) > P(C) > P(B)$

38. What is the value of the given series $\frac{1}{3} + \frac{1}{10} + \frac{1}{21} + \frac{1}{36} + \dots$

- (a) $1 - 2 \log_e 2$ (b) $2 - \log_e 4$ (c) $1 - \log_e 2$ (d) $2 - 2 \log_e 2$

39. For each x in $[0, 1]$, define

$$\begin{aligned} f(x) &= 2x, & \text{if } 0 \leq x \leq \frac{1}{2}; \\ f(x) &= 2 - 2x, & \text{if } \frac{1}{2} < x \leq 1. \end{aligned}$$

Let $f^{[2]}(x) = f(f(x))$, and $f^{[n+1]}(x) = f^{[n]}(f(x))$ for each integer $n \geq 2$. For how many values of x in $[0, 1]$ is $f^{[2017]}(x) = \frac{1}{2}$?

- (a) 0 (b) 4034 (c) 2017^2 (d) 2^{2017}

40. Which of the following is/are true?

- (a) There exists an uncountable subset of \mathbb{Q}
(b) There exists a countable subset of \mathbb{Q}
(c) There exists a countable subset of \mathbb{N}
(d) There exists an uncountable subset of \mathbb{R}

Tie-Breaker Question

41. (I) What is the maximum number of rooks that one place on an 8×8 chessboard such that no two rooks can hit each other?

(II) How many bishops can one put on an 8×8 chessboard such that no two bishops can hit each other?

- (a) (I)-8,(II)-14;
(b) (I)-12,(II)-12;
(c) (I)-8,(II)-10;
(d) (I)-10,(II)-10;
-

Space for Rough Work

Space for Rough Work