Additions to
Linear Functional Analysisfor Scientists and Engineers

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In the following, **p. i, +j** means the jth line from the top on page i, whereas **p. i, −j** means the jth line from the bottom on page i.

Chapter 2

p. 37, +15, +16

Thus we obtain $C(E) \subset L^\infty(E) \subset L^2(E) \subset L^1(E)$ as opposed to $c_{00} \subset \ell^1 \subset \ell^2 \subset \ell^\infty$.

Chapter 3

p. 92, −5

For $p \in \mathcal{P}$ and $x, y \in X$,

$$|p(x) - p(y)| \leq p(x-y) \leq \wp(x-y) \leq \alpha\|x-y\|.$$

Now given $\epsilon > 0$, if we let $\delta := \epsilon/\alpha$, then $|p(x) - p(y)| < \epsilon$ for all $p \in \mathcal{P}$, and $x, y \in X$ with $\|x - y\| < \delta$. Hence the set $\mathcal{P}$ is equicontinuous on X.

p. 98, +1

The first assertion in Remark 3.29 is illustrated by Example 3.19 (i). Note that the map $F : C^1([0,1]) \to C([0,1])$ given by $F(x) := x'$ is a closed map. Another illustration of this assertion is given by the following example.

Let $X := c_{00}$ with the sup norm $\|\cdot\|_\infty$, and for $x \in X$, let $p(x) := \sum_{k=1}^\infty |x(k)|$. Then the seminorm p on X is discontinuous, even though it is countably subadditive. To see this, let $x_n := e_1 + \cdots + e_n$ for $n \in \mathbb{N}$. Since $\|x_n\|_\infty = 1$ for all $n \in \mathbb{N}$, and $p(x_n) = n \to \infty$, we see that p is discontinuous on X. Next, let $Y := \ell^1$ with the usual norm $\|\cdot\|_1$. Then Y is a Banach space. Define $F : X \to Y$ by $F(x) := x$ for $x \in X$. To that F is a closed map, we note that if $x_n \to x$ in X, and $F(x_n) \to y$ in Y, then

$$y(j) = \lim_{j\to\infty} F(x_n)(j) = \lim_{j\to\infty} x_n(j) = x(j) \text{ for each } j \in \mathbb{N},$$

and so $y = x = F(x)$. As a result, the seminorm p is countably subadditive on X. Note that in this example, Y is a Banach space, but X is not.

p. 98, +3

A proof of the converse statement in Remark 3.29 was given by Zabreiko in an email correspondence. It is as follows. Suppose the seminorm p is countably subadditive. To prove that the linear map F is closed, we let $x_n \to 0$ in X such that $F(x_n) \to y$ in Y, and show that $y = 0$. There are $n_1 < n_2 < \cdots$ in $\mathbb{N}$ such that $\|F(x_{n_k}) - y\| \le 1/2^k$ for each $k \in \mathbb{N}$. We can, therefore, assume without loss of generality, that $\sum_{k=1}^{\infty} \|F(x_k) - y\| < \infty$. Fix $n \in \mathbb{N}$, and let $m \ge n$. Then

$$\sum_{k=n}^{m} (x_k - x_{k+1}) = x_n - x_{m+1} \to x_n \text{ as } m \to \infty, \text{ and so } x_n = \sum_{k=n}^{\infty} (x_k - x_{k+1}).$$

Since p is assumed to be countably subadditive and since F is linear,

$$\|F(x_n)\| = p(x_n) \le \sum_{k=n}^{m} p(x_k - x_{k+1}) = \sum_{k=n}^{m} \|F(x_k) - F(x_{k+1})\|.$$

By the triangle inequality,

$$\|F(x_n)\| \le \sum_{k=n}^{m} \|F(x_k) - y\| + \sum_{k=n}^{m} \|y - F(x_{k+1})\|.$$

Also, since $\sum_{k=1}^{\infty} \|F(x_k) - y\| < \infty$, we see that

$$\|F(x_n)\| \le \sum_{k=n}^{\infty} \|F(x_k) - y\| + \sum_{k=n}^{\infty} \|y - F(x_{k+1})\| \to 0 + 0 = 0 \text{ as } n \to \infty,$$

This shows that $F(x_n) \to 0$. But $F(x_n) \to y$, and so $y = 0$. It is clear that neither X nor Y is assumed to be a Banach space.

Chapter 5

p. 167, +4

As a result, $\|(I - A)^{-1}\| \le \sum_{n=0}^{\infty} \|A^n\| \le \sum_{n=0}^{\infty} \|A\|^n = 1/(1 - \|A\|)$.