Preface

The aim of this book is to provide a short and simple introduction to the charming subject of linear functional analysis. The adjective 'linear' is used to indicate our focus on linear maps between linear spaces. The applicability of the topics covered to problems in science and engineering is kept in mind.

In principle, this book is accessible to anyone who has completed a course in linear algebra and a course in real analysis. Relevant topics from these subjects are collated in Chap. 1 for a ready reference. A familiarity with measure theory is not required for the development of the main results in this book, but it would help appreciate them better. For this reason, a sketch of rudimentary results about the Lebesgue measure on the real line is included in Chap. 1. To keep the prerequisites minimal, we have restricted to metric topology, and to the Lebesgue measure, that is, neither general topological spaces nor arbitrary measure spaces would be considered.

Year after year, several students from the engineering branches, who took my course on functional analysis at the Indian Institute of Technology Bombay, have exclaimed 'Why were we not introduced to this subject in the early stages of our work? Many things with which we had to struggle hard would have fallen in place right at the start.' Such comments have prompted me to write this book. It is also suitable for an introductory course in functional analysis as a part of a masters' program in mathematics.

The treatment of each topic is brief, and to the point. This book is not meant to be a compendium of all the relevant results in a given topic. On the other hand, we give a lot of examples of results proved in the book. The entire book can be followed as a text for a course in linear functional analysis without having to make any specific selection of topics treated here. Teaching courses in analysis for decades has convinced me that if a student correctly grasps the notion of convergence of a sequence in a metric space, then (s)he can sail through the course easily. We define most concepts in metric topology in terms of sequences; these include total boundedness, compactness and uniform continuity. Also, among the ℓ^p and L^p

spaces, $1 \leq p \leq \infty$, we consider only the cases $p = 1, 2$ and ∞, since they represent all essential features, and since they can be treated easily.

A novelty of this book is the inclusion of a result of Zabreiko which states that every countably subadditive seminorm on a Banach space is continuous. Once this result is proved using the Baire theorem, several important theorems can be deduced quickly. They include the uniform boundedness principle, the closed graph theorem, the bounded inverse theorem and the open mapping theorem. Surprisingly, not many textbooks on functional analysis have followed this efficient path. Another noteworthy feature of this book is that the spectral theory is treated at a single location. It deals with the eigenspectrum, the approximate eigenspectrum and the spectrum of a bounded operator, of a compact operator and of their transposes and adjoints. The spectral theorem gives a characterization of a compact self-adjoint operator.

The main body of this book consists of Chaps. 2–5, each of which is divided into four sections. Forty exercises, roughly ten on each section, are given at the end of each of these chapters. They are of varying levels of difficulty; some of them are meant to reassure the reader that (s)he has indeed grasped the core ideas developed in the chapter, while others demand some ingenuity on part of the reader. The exercises follow the same order as the text on which they are based. All exercises are in the form of statements to be justified. Their solutions are given at the end.

When a new concept is introduced, it appears in boldface. The symbol $:=$ is used for defining various objects. Definitions are not numbered, but can be located easily by using the index given at the end. All other things (lemmas, propositions, theorems, remarks, examples) are numbered serially in each chapter, and so are the exercises on it. Before the chapters begin, a list of symbols and abbreviations (along with their descriptions and the page numbers where they appear for the first time) is given in the order in which they appear in the text.

The Department of Mathematics of the Indian Institute of Technology Bombay deserves credit for providing excellent infrastructure. I thank Peeter Oja for bringing the Zabreiko theorem to my attention. I am indebted to Ameer Athavale, Anjan Chakrabarty and Venkitesh Iyer for critically reading the book and making useful suggestions. I am grateful to my wife Nirmala Limaye for her wholehearted support, both material and moral. She has drawn all the figures in this book using PSTricks, and has also figured out, without using personal tricks, how to keep me in good shape.

I would appreciate receiving comments, suggestions and corrections. A dynamic errata together with all relevant information about this book will be posted at http://www.math.iitb.ac.in/~bvl/. I encourage readers to visit this webpage for updates concerning this book.

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