Conference on Functional Analysis

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Titles & Abstracts

Department of Mathematics
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Search of fine accuracy order in B-spline collocation

Athira Babu
Cochin University

Abstract: It is highly important to maintain the smoothness of B-splines for better approximation accuracy when we try to approximate an unknown function using B-spline collocation. Let [a, b] be the spacial domain of the differential equation we concern, subdivide the domain into N subintervals of same length $\frac{b-a}{N}$ having the node points a = $x_0 < x_1 < x_2 \cdots < x_N$ = b. Denote by $B_j(x)$, the B-spline symmetric about x_j and supported in the interval $[x_{j-2}, x_{j+2})$. We write the unknown solution as $U_N(x,t) = \sum_{j=-1}^{N+1} d_j(t)B_j(x)$, where $d_j(t)$ are the coefficient functions of t. To remove the undefined nodes in the expansion we need to modify the cubic spline so that the approximation becomes $U_N(x,t) = \sum_{j=0}^{N} d_j(t)\tilde{B}_j(x)$. The accuracy order of cubic B-spline is the highest m such that 1, x, x^2, $x^3 \cdots x^{m-1}$ can be written as a linear combination of these $B_j(x)$ in the domain. Polynomial space P_3(x) can be generated in the domain with $B_j(x)$ for j = -1, 0, 1, 2, $\cdots$ N+1, that is having accuracy order 4, but our intension is to modify splines without diminishing the accuracy order. The literature describes the modification having accuracy order 2. We are seeking whether it is possible a modification which has an accuracy order more than two.

Isometric dilations and von Neumann inequality

Sibaprasad Barik
IIT Bombay

Abstract: It is well known that for an arbitrary n-tuple of commuting contractions, $n \geq 3$, neither the existence of isometric dilation nor the von Neumann inequality holds. This talk includes explicit isometric dilations for a large class of n-tuple ($n \geq 3$) of commuting contractions. Isometric dilations, in our context, heavily rely on a well-known commutant lifting theorem on the polydisc by Ball, Li, Timotin and Trent. We will see that a refined von Neumann inequality, in terms of an algebraic variety in the closure of the polydisc in $\mathbb{C}^n$, holds for n-tuples in the class. This is a joint work with B. Krishna Das and J. Sarkar.

On the quasi-spilt property and commutant of the generator of q-deformed Araki-Woods von Neumann algebras

Panchugopal Bikram
NISER Bhubaneswar

Abstract: In this talk we discuss split (quasi) inclusions of von Neumann algebras that will be used as an auxiliary tool to investigate the q-deformed Araki-Woods von Neumann algebras. Prove that the generating abelian subalgebras arsing from vectors in the ergodic component of Hiais construction of the q-deformed Araki-Woods von Neumann algebras are quasi-split and has large relative commutant. This is a joint work with K. Mukhrejee.

Roots of completely positive maps

B.V.R. Bhat
ISI Bangalore

Abstract: We introduce the concept of completely positive roots of completely positive maps on operator algebras. We do this in different forms: as asymptotic roots, proper discrete roots and in continuous one parameter setting.

We understand the situation better for states. Here is the precise question: Given a state ϕ on a unital C^*algebra $\mathcal{A}$ we look at unital quantum dynamical semigroups τ_t on $\mathcal{A}$ such that $\tau_{t_0}(\cdot) = \phi(\cdot)I$ for some $t_0 > 0$. We see that for the von Neumann algebra $\mathcal{B}(\mathcal{H})$, such quantum dynamical semigroups dilate to semigroups of unital endomorphisms (E_0-semigroups) in standard form and conversely all E_0-semigroups in standard form arise this way.

We present several general existence and non-existence results, some special examples in settings where we understand the situation better, and several challenging open problems. This is based on a joint work with Robin Hillier, Nirupama Mallick and Vijaya Kumar U.

Factors of hypercontractions

Monojit Bhattacharjee
IIT Bombay

Abstract: In this talk, we discuss about a class of contractive factors of m-hypercontractions for $m \in \mathbb{N}$. We obtain a characterization of such factors and this is achieved by finding explicit dilations of these factors on certain weighted Bergman spaces. This is a generalization of the work done in Factorizations of Contractions by Das, Sarkar and Sarkar. This is a joint work with B. Krishna Das.

Classification of crossed products of rotation algebras by cyclic subgroups of $SL_2(\mathbb{Z})$

Sayan Chakraborty
IISER Bhopal

Abstract: Let θ be a real number and A be a matrix in $SL_2(\mathbb{Z})$ of infinite order. We compute the K-theory of the crossed product C*-algebra $A_\theta \rtimes_A \mathbb{Z}$, where A_θ is the rotation algebra. From the classification theory of C*-algebras we also deduce the isomorphism classes (depending on θ and A) of $A_\theta \rtimes_A \mathbb{Z}$. This is an extension of similar results obtained for A_θ by Elliott and for $A_\theta \rtimes_A \mathbb{Z}$ (when θ is irrational) by Chakraborty, Boenicke, He, and Liao.

This is a joint work with Boenicke (Glasgow), He (Tokyo), and Liao (Ottawa).

Classifiation of rank 3 homogeneous operators

Prahllad Deb
IISER Kolkata

Abstract: A bounded linear operator T on a Hilbert space $\mathcal{H}$ is said to be homogeneous with respect to a subgroup G of the automorphism group of some domain $\Omega \subset \mathbb{C}^n$ if the spectrum of T is contained in Ω as well as, for every $g \in G$, $g(T)$ is unitarily equivalent to T. For $\Omega = \mathbb{D}$, all operators in the Cowen-Douglas class $B_k(\mathbb{D})$ of rank k, $k \geq 1$, homogeneous with respect to the automorphism group of $\mathbb{D}$ are classified by A. Korányi and G. Misra. So it leads to study tuples of operators in $B_k(\mathbb{D}^n)$, $n \geq 2$, $k \geq 1$, which are homogeneous with respect to some closed subgroup of the automorphism group of $\mathbb{D}^n$. All operators in $B_k(\mathbb{D}^n)$, which are homogeneous with respect to either the group

$\mathrm{Aut}(\mathbb{D})^n$, or, $\mathrm{Aut}(\mathbb{D}^n)$, are known for $k = 1, 2$. In this talk, we describe all irreducible tuple of operators (upto unitary equivalence) in $B_3(\mathbb{D}^n)$ which are homogeneous with respect to the subgroup $\mathrm{Aut}(\mathbb{D})^n$ of the automorphism group of $\mathbb{D}^n$. Moreover, for $n = 2$, we show that a subclass of operator tuples in $B_3(\mathbb{D}^n)$ are also homogeneous with respect to the full automorphism group of $\mathbb{D}^n$ while, for $n > 2$, every tuple of operators in $B_3(\mathbb{D}^n)$ fails to be homogeneous with respect to $\mathrm{Aut}(\mathbb{D}^n)$. This is a joint work with Somnath Hazra.

Factorizations of Schur functions

Ramlal Debnath
ISI Bangalore

Abstract: The Schur class, denoted by $\mathcal{S}(\mathbb{D})$, is the set of all functions analytic and bounded by one in modulus in the open unit disc $\mathbb{D}$ in the complex plane $\mathbb{C}$, that is

$$\mathcal{S}(\mathbb{D}) = \{\varphi \in H^\infty(\mathbb{D}) : \|\varphi\|_\infty := \sup_{z\in\mathbb{D}} |\varphi(z)| \leq 1\}.$$

The elements of $\mathcal{S}(\mathbb{D})$ are called Schur functions.

Suppose $\varphi : \mathbb{D} \to \mathbb{C}$ is a function. A classical result going back to I. Schur states: $\varphi \in \mathcal{S}(\mathbb{D})$ if and only if there exist a Hilbert space $\mathcal{H}$ and an isometry (known as colligation matrix or scattering matrix and nonunique in general)

$$V = \begin{bmatrix} a & B \\ C & D \end{bmatrix} : \mathbb{C} \oplus \mathcal{H} \to \mathbb{C} \oplus \mathcal{H},$$

such that φ admits a transfer function realization corresponding to V, that is

$$\varphi(z) = a + zB(I_\mathcal{H} - zD)^{-1}C \qquad (z \in \mathbb{D}).$$

An analogous statement holds true for Schur functions on the bidisc. On the other hand, Schur-Agler class functions in several variables is a well-known "analogue" of Schur functions on $\mathbb{D}$. In this talk, we will discuss algorithms to factorize bounded analytic functions and Schur-Agler class of functions in terms of colligation matrices. This is joint work with Jaydeb Sarkar.

The orbit of a bounded operator under the Möbius group modulo similarity equivalence

Soumitra Ghara
IISc Bangalore

Abstract: Let Möb denote the group of biholomorphic automorphisms of the unit disc, and let $(\mathrm{Möb} \cdot T)$ be the orbit of a Hilbert space operator T under the action of Möb. If the quotient $(\mathrm{Möb} \cdot T)/\sim$, where $\sim$ is the similarity between two operators is a singleton, then the operator T is said to be weakly homogeneous. In this talk, we will discuss a criterion to determine if the operator M_z of multiplication by the coordinate function z on a reproducing kernel Hilbert space is weakly homogeneous. We will use this to show that there exists a Möbius bounded weakly homogeneous operator which is not similar to any homogeneous operator, answering a question of Bagchi and Misra in the negative.

Quantum isometry group of the odd dimensional quantum spheres

D. Goswami
ISI Kolkata

Abstract: We will compute the universal compact type Hopf algebra Co acting on the algebra of odd dimensional quantum sphere such the the coaction preserves the natural set of generators and the canonical invariant functional on the q sphere. This is identified with the Hopf algebra underlying the corresponding quantum unitary group. An analytic version of this result in the framework of quantum isometry group of spectral triples has also been established. Based on a joint paper with Suvrajit Bhattacharjee.

An analogue of Hall's Marriage Theorem for regular subfactors

Ved Gupta
JNU

Abstract: Given a subgroup H of a finite group G, as an application of the well known Hall's Marriage Theorem, it is known that there exists a finite set in G which acts simultaneously as a set of representatives for the left cosets as well as the right cosets of H in G.

Given an inclusion of type II_1 factors $N \subset M$ with finite Jones' index, M can be treated naturally as a left and a right N-module. Pimsner and Popa showed that M is finitely generated as a left (equivalently, right) N-module. However, it is not yet known whether a common set can act simultaneously as a left and a right generating set. We answer this question in the affirmative for the so called regular subfactors.

This talk is based on a joint work with Keshab Chandra Bakshi titled "On orthogonal systems, two-sided bases and regular subfactors".

Isometric Dilations and von Neumann inequality for Operator Tuples

Kalpesh J. Haria
IIT Mandi

Abstract: The celebrated Sz.-Nagy and Foias and Ando theorems state that a single contraction, or a pair of commuting contractions, acting on a Hilbert space always possesses isometric dilation and subsequently satisfies the von Neumann inequality for polynomials in $\mathbb{C}[z]$ or $\mathbb{C}[z_1, z_2]$, respectively. However, in general, neither the existence of isometric dilation nor the von Neumann inequality holds for n-tuples, $n \geq 3$, of commuting contractions. In this talk, we shall provide a taste of isometric dilations, von Neumann inequality and a refined version of von Neumann inequality for a large class of n-tuples, $n \geq 3$, of commuting contractions.
This is a joint work with Sibaprasad Barik, B. Krishna Das, and Jaydeb Sarkar.

Representation and topological correspondences of Fell bundles

Rohit Dilip Holkar
IISER Bhopal

Abstract: Fell bundles are, possibly, the most generalised form of C*-dynamical systems. Fell bundles include global actions of groups or groupoids on C*-algebras by automorphisms, twisted continuous actions, as well as, twisted partial actions. Muhly and Williams defined the C*-algebra of a saturated Fell bundle for the first time (2008) using disintegration techniques of Renault. They also defined the equivalence of Fell bundles. However, unsaturated Fell bundles have not yet been treated even though they

appear naturally (e.g. a Hilbert C*-module that is not full). We define a universal property for the full C*-algebra of unsaturated Fell bundles over locally compact Hausdorff groupoids. With the help of this, we define the full C*-algebra of a Fell bundle. If time permits, we shall also discuss the topological correspondences of Fell bundles. This is a joint work with Alcides Buss and Ralf Meyer.

A model for non-commutative vector lattices

Anil Kumar Karn
NISER Bhubaneswar

Abstract: In this talk, we shall discuss an order theoretic aspect of the algebraic orthogonality in C*-algebras and based on the outcome of the observations, we shall discuss a model for non-commutative vector lattices which includes vector lattices as well as the self-adjoint parts of C*-algebras.

Loewner's necessary condition on entrywise preservers in fixed dimension

Apoorva Khare
IISc Bangalore

Abstract: The question of which entrywise maps preserve positive semidefiniteness in a fixed dimension N is a challenging one, with all cases $N > 2$ open to date. (The $N = 2$ case was worked out by Vasudeva.) For dimension $N > 2$, in a sense the only known result for arbitrary functions is a classical one by Charles Loewner, which appears in a 1969 paper by (his student) Roger A. Horn.

In this talk, we will focus on the proof of the Horn–Loewner theorem. Some highlights along the way are:

(a) Friedrich mollifiers (1944) on the line,
(b) the above characterization for $2x2$ matrices by Harkrishan L. Vasudeva (1979),
(c) a remarkable "converse mean-value theorem" by Ralph P. Boas and David V. Widder (1940), and
(d) a Banach space variant of a classical result by Alexander M. Ostrowski (1929).

If time permits, I will end by extending a computation by Loewner inside his proof, using which I showed (2018) that all Schur polynomials (a class of symmetric, homogeneous algebraic functions) emerge naturally out of a determinant involving an arbitrary smooth function.

Direct limit of absolute matrix order unit spaces

Amit Kumar
NISER Bhubaneswar

Abstract: In this talk, we introduce the notions of absolutely matrix ordered spaces and absolute matrix order unit spaces in the context of matrix ordered spaces. We also study some properties of absolutely matrix ordered spaces and absolute matrix order unit spaces. Later we shall find out the direct limit of "Absolute Matrix Order Unit spaces".

Weighted composition operators

Romesh Kumar
Jammu University

Abstract: TBA

Operator-valued multishifts with invertible operator weights

Surjit Kumar
IISc Bangalore

Abstract: Let $\mathbb{C}[z_1, \cdots, z_d]$ denote the ring of complex valued polynomials in d variables equipped with the supremum norm on unit polydisc. Let $T = (T_1, \ldots, T_d)$ be a d-tuple of commuting operators on a Hilbert space $\mathcal{H}$. The d-tuple of commuting operators T induces a homomorphism $\rho_T : p \to p(T)$ for all $p \in \mathbb{C}[z_1, \cdots, z_d]$.

Recently, Hartz proved that the induced homomorphism ρ_T of a commuting contractive classical multishift T with non-zero weights is completely contractive, and in particular, it satisfies von Neumanns inequality. This result does not extend to the class of commuting operator-valued multishifts with invertible operator weights. In fact, the induced homomorphism is not even contractive. We also exhibit several families of operator-valued multishifts for which the von Neumann's inequality always holds. Similarity and unitary equivalence of operator-valued multishifts with invertible operator weights will be also discussed. Finally, we discuss function theoretic behaviour such as the set Ω of points where evaluation is bounded and the reproducing kernel on Ω of operator-valued multishifts with invertible operator weights.

On weighted browder spectrum

Sarita Kumari
Delhi University

Abstract: The main results in the present paper at extending some properties of α- Fredholm operators to α- Browder operators. The notion of α-B-Fredholm operator, α-B-Browder operator and α-Drazin invertibility have been introduced and the relation between these operators and their corresponding spectra has been investigated.

Some problems on approximation of functions

Vishnu Narayan Mishra
Indira Gandhi National Tribal University

Abstract: In this talk, we discuss the degree of approximation of functions using various types of summability transforms methods in different spaces. During this talk, few applications of approximations of functions will also be highlighted.

TBA

G. Misra
IISc Bangalore

Abstract: TBA

K-homogeneous operators on bounded symmetric domain of type-I

Paramita Pramanick
IISc Bangalore

Abstract: Let Ω be the type-I bounded symmetric domain and K be the maximal compact subgroup of $\mathrm{Aut}(\Omega)$. A commuting d-tuple of operators $T = (T_1, \ldots, T_d)$ is said to be K-homogeneous if $k \cdot T$, which is defined via the usual functional calculus, is unitarily equivalent to T. In this talk, after making certain assumptions on a commuting tuple of operators T, I am going to discuss a classification of K-homogeneous operator tuples among these. Various other properties of these operator tuples like boundedness, similarity are also be discussed.

Korovkin -type Theorems for Toeplitz Operators

Rahul Rajan
Cochin University

Abstract: The classical Korovkin theorem due to P.P Korovkin [1] states the following. Let $\{\Phi_n\}$ be a sequence of positive linear maps on $C[0,1]$. If

$$\Phi_n(f) \to f \text{for every } f \text{ in the set} \{1, x, x^2\},$$

then

$$\Phi_n(f) \to f \text{for every } f \text{ in } C[0,1].$$

Here the convergence is the uniform convergence of sequence of functions. There are several variations of these results into various settings such as Banach algebras, C^*- algebras etc. In 1999, Stefano Serra Cappizano obtained some Korovkin -type results for obtaining optimal Preconditioners for solving linear systems with Toeplitz structure [2]. The result is for self -adjoint Toeplitz matrices of growing order (they are truncations of Toeplitz operators on the Hardy space of unit circle) which is generated using the Fourier coefficients of a single real valued function $f \in L^\infty(\mathbb{T})$. The test set was the set of all trigonometric polynomials and the notion of convergence is in the sense of eigenvalue clustering of sequences of Hermitian matrices of growing order. Assuming the convergence for trigonometric polynomials, he could prove that the convergence holds for Toeplitz Operators with symbols from the C^*- algebra of functions generated by the test set.

In [3], these notions were generalized into the setting of operators acting on infinite dimensional Hilbert spaces. It generalized and improved the existing results. Recently, we extended these results to the non self- adjoint Toeplitz matrices [4]. The results were extended to Toeplitz operators with continuous symbols. Presently, we consider similar results for Toeplitz operators on Bergman Spaces, Fock spaces etc. We have obtained some similar results for Toeplitz operators on Bergman space with symbols continuous on closed disk. We could prove that the convergence will hold for the Toeplitz Operators with symbols from C^*- algebra generated by the test set and also for the Operators in the Operator C^*- algebra generated by the Toeplitz Operators with the symbols from the test set. We try to extend these results to Toeplitz operators corresponding to a large class of symbols containing the continuous symbols. In the talk, we briefly present these developments. This is a joint work with Prof. Wolfram Bauer, Institute of Analysis, Leibniz University of Hannover, Germany.

References

[1] Korovkin, P. P. *Linear operators and approximation theory.* (1960).

[2] Serra- Capizzano, S. *A Korovkin-type theory for finite Toeplitz operators via matrix algebras.* Numer. Math. 82 (1999), no. 1, 117–142.
[3] Kiran Kumar, V. B; Namboodiri, M. N. N.; Serra-Capizzano, S. *Preconditioners and Korovkin-type theorems for infinite-dimensional bounded linear operators via completely positive maps.* Studia Math. **218** (2013), no. 2, 95–118.
[4] Kiran Kumar, V. B.; Namboodiri, M. N. N.; Rajan, Rahul. *A Korovkin-type theory for non-self-adjoint Toeplitz operators.* Linear Algebra Appl. 543 (2018), 140–161.

Hausdorff Moment Sequences induced by rational functions

Md. Ramiz Reza
IIT Kanpur

Abstract: Given a sequence $(x_n)_{n\in\mathbb{Z}_+}$ of positive numbers, Hausdorff moment problem asks what are the conditions on the sequence so that there exist a positive Radon measure μ supported on the interval $[0,1]$ such that

$$x_n = \int_0^1 t^n d\mu(t), \;\; n \in \mathbb{Z}_+.$$

We study the Hausdorff moment problem for a class of sequences, namely $(r(n))_{n\in\mathbb{Z}_+}$, where r is a rational function in the complex plane. We try to find conditions on the location of the zeros and poles of a rational function r so that $(r(n))_{n\in\mathbb{Z}_+}$ becomes a Hausdorff moment sequence. We obtain a necessary condition for such sequence to be a Hausdorff moment sequence. We found an interesting connection between Hausdorff moment problem for this class of sequences with finite divided differences and convolution of complex exponential functions. G. Misra asked whether the module tensor product of a subnormal module with the Hardy module over the polynomial ring is again a subnormal module or not. Using our necessary condition we answer the question of G. Misra in negative. Finally, we obtain a complete characterization of all real polynomials p of degree up to 4 and a certain class of real polynomials of degree 5 for which the sequence $(1/p(n))_{n\in\mathbb{Z}_+}$ is a Hausdorff moment sequence.

Quantum $E(2)$ groups for complex deformation parameter

Sutanu Roy
NISER Bhubaneswar

Abstract: Quantum $E(2)$ groups for (non-zero) real deformation parameter is the first and extensively studied example of a locally compact quantum group (lcqg) constructed by S. L. Woronowicz in the early nineties. However, it fails to be a lcqg for (non-zero) complex deformation parameters. In this talk we shall address this problem and show that it becomes a lcqg in a suitable monoidal category of C^*-algebras, or, in short, a braided quantum group. Furthermore, we shall also discuss the contraction procedure for quantum $SU(2)$ and $E(2)$ for complex deformation parameter. This is a joint work in progress with Atibur Rahaman.

Dilation theory on the bidisk revisited

Haripada Sau
IIT Guwahati

Abstract: Arguably, Ando's proof of his dilation theorem has proven to be of limited utility for understanding a geometric structure of the dilation (a consequence of the lack of uniquenss for minimal Ando dilations), and consequently there has been some investment in the use of other approaches; see Das-Sarkar-Sarkar, Adv. Math., 2017 and references therein for analysis in some special cases. In this talk we shall see how the successful dilation/model theory of Sz.-Nagy and Foias proves to be a useful tool in giving a classification of Ando dilations; this leads to two new geometric proofs of the Ando dilation theorem. This is a joint work with Professor Joseph A Ball of Virginia Tech.

Trace results for multi-variable operator families

K.B. Sinha
JNCASR

Abstract: The Berger-Shaw theorem for a hyponormal operator connects this property (along with an assumption of finite multiplicity) "somewhat mysteriously" with trace property. What could be the corresponding idea for 2 or more commuting tuples $T = \{T_j\}$, which may lead to similar trace results? In this talk this will be explored; an "operator- determinant of T" will be defined, for which a trace result follows, i.e. if the operator-determinant is positive and if some assumptions along with that of finite multiplicity are made, then it is trace-class, extending the Berger-Shaw theorem for single operator (This work is jointly with P. Pramanick and G. Misra).

CCR and CAR flows over convex cones

R. Srinivasan
CMI

Abstract: I will discuss the problem of classifying CCR flows and CAR flows over convex cones, up to cocycle conjugacy.

Generating wandering subspaces for doubly commuting covariant representations

S. Veerabathiran
Madras University

Abstract: We obtain a Halmos-Richter-type wandering subspace theorem for covariant representations of C^*-correspondences. Further the notion of Cauchy dual and a version of Shimorins Wold-type decomposition for covariant representations of C^*-correspondences is explored and as an application a wandering subspace theorem for doubly commuting covariant representations is derived. Using this wandering subspace theorem generating wandering subspaces are characterized for covariant representations of product systems in terms of the doubly commutativity condition.