Course Curricula : M.Sc. Mathematics

First Semester						Second Semester					
Course	**Name**	**L**	**T**	**P**	**C**	**Course**	**Name**	**L**	**T**	**P**	**C**
CS101	Computer Programming & Utilization	2	0	2	6	MA 406	General Topology	3	1	0	8
MA 401	Linear Algebra	3	1	0	8	MA 408	Measure Theory	3	1	0	8
MA 403	Real Analysis	3	1	0	8	MA 410	Multivariable Calculus	2	1	0	6
MA 417	Ordinary Differential Equations	3	1	0	8	MA 412	Complex Analysis	3	1	0	8
MA 419	Basic Algebra	3	1	0	8	MA 414	Algebra I	3	1	0	8
	Total Credits	**14**	**4**	**2**	**38**		**Total Credits**	**14**	**5**	**0**	**38**

Third Semester						Fourth semester					
MA 503	Functional Analysis	3	1	0	8	ES 200/	Environmental Science/	--	--	--	6
MA 515	Partial differential Equations	3	1	0	8	HS 200	Dept. Elective/Institute Elective				
	Elective I	2	1	0	6		Elective IV	2	1	0	6
	Elective II	2	1	0	6		Elective V	2	1	0	6
	Elective III	2	1	0	6		Elective VI	2	1	0	6
MA 593	Project I (Optional)	-	-	-	4		Elective VII	2	1	0	6
						MA 598	Project II/Dept. Elective/Institute Elective	-	-	-	6
	Total Credits	**12**	**5**	**0**	**34**		**Total Credits**	**11**	**4**	**0**	**36**
	Grand Total							**51**	**18**	**2**	**140**

Electives I – III						Electives IV – VII					
MA 521	Theory of Analytic Functions	2	1	0	6	MA 504	Operators on Hilbert Spaces	2	1	0	6
MA 523	Basic Number Theory	2	1	0	6	MA 510	Introduction to Algebraic Geometry	2	1	0	6
MA 525	Dynamical Systems	2	1	0	6	MA 518	Spectral Approximation	2	1	0	6
SI 537	Probability II	3	1	0	8	MA 524	Algebraic Number Theory	2	1	0	6
MA 538	Representation Theory of Finite Groups	2	1	0	6	MA 526	Commutative Algebra	2	1	0	6
MA 539	Spline Theory and Variational Methods	2	1	0	6	MA 528	Hyperplane Arrangements	3	0	0	6
MA 556	Differential Geometry	2	1	0	6	MA 530	Nonlinear Analysis	2	1	0	6
MA 581	Elements of Differential Topology	2	1	0	6	MA 532	Analytic Number Theory	2	1	0	6
SI 507	Numerical Analysis	3	1	0	8	MA 534	Modern Theory of PDE	2	1	0	6
MA5101	Algebra II	2	1	0	6	MA 540	Numerical Methods for PDE	2	1	0	6
MA5103	Algebraic Combinatorics	2	1	0	6	MA 562	Mathematical Theory of Finite Elements	2	1	0	6
MA5105	Coding Theory	2	1	0	6	SI 416	Optimization	2	1	0	6
MA5107	Continuum Mechanics	2	1	0	6	SI 527	Introduction to Derivative Pricing	2	1	0	6
MA5109	Graph Theory	2	1	0	6	MA5102	Basic Algebraic Topology	2	1	0	6
MA5111	Theory of Finite Semigroups	2	1	0	6	MA5104	Hyperbolic Conservation Laws	2	1	0	6
MA5113	Category Theory I	3	0	0	6	MA5106	Introduction to Fourier Analysis	2	1	0	6
MA5115	Hopf Algebras	2	1	0	6	MA5108	Lie Groups and Lie Algebra	2	1	0	6
						MA5110	Noncommutative Algebra	2	1	0	6
						MA5112	Introduction to Math. Methods	2	1	0	6
						MA5116	Species and Operads	2	1	0	6
						MA5118	Category Theory II	3	0	0	6

COURSE CONTENTS

CS 101 Computer Programming & Utilization 2 0 2 6

Functional organization of computers, algorithms, basic programming concepts, FORTRAN language programming. Program testing and debugging, Modular programming subroutines: Selected examples from Numerical Analysis, Game playing, sorting/ searching methods, etc.

Texts / References:

N.N. Biswas, FORTRAN IV Computer Programming, Radiant Books, 1979.

K.D. Sharma, Programming in Fortran IV, Affiliated East West, 1976.

ES 200 Environmental Studies 3 0 0 3

Multidisciplinary nature of environmental problems; Ecosystems, Biodiversity and its conservation; Indicators of environmental pollution; Environment and human health; Utilization of natural resources and environmental degradation. Sustainable development; Environmental policy and law; Environmental impact assessment; Pollution of lakes, rivers and groundwater. Principles of water and wastewater treatment; Solid and hazardous waste management. Air Pollution: sources and effects, Atmospheric transport of pollutants; Noise pollution; Global issues and climate change: Global warming, Acid rain, Ozone layer depletion.

Texts / References:

W. P. Cunningham and M. A. Cunningham, Principles of Environmental Science, Tata McGraw-Hill Publishing Company, 2002.

J. A. Nathanson, Basic Environmental Technology: Water Supply Waste Management and Pollution Control, 4th Edition, Prentice Hall of India, 2002.

G.M. Masters, Introduction to Environmental Engineering and Science, Second Indian Reprint, Prentice-Hall of India, 2004.

M. L. Davis and D. A. Cornwell, Introduction to Environmental Engineering, 2nd Edition**,** McGraw Hill, 1998.

R. T. Wright, Environmental Science: Towards a Sustainable Future, 9th Edition, Prentice Hall of India, 2007.

Supplementary Reading Materials (Selected Book Chapters and Papers)

HS 200 Environmental Studies 3 0 0 3

Social issues and the environment, Public awareness and Human rights, Indicators of sustainability, Governance of Natural Resources - Common pool resources: issues and management.

Environmental ethics, Religion and environment, Wilderness and Developing Trends, Environmental movements and Activism, Social Ecology and Bioregionalism, Environmental justice.

Environmental economics, Trade and environment, Economics of environmental regulation, Natural resource accounting, Green GDP.

Environment and development, Resettlement and rehabilitation of people, Impacts of climate change on economy and society, Vulnerability and adaptation to climate change.

Text / References:

N. Agar, Life's Intrinsic Value, Columbia University Press, 2001.

P. Dasgupta and G. Maler, G. (Eds.), The Environment and Emerging Development Issues, Vol. I, Oxford University Press, 1997.

R. Guha, “Mahatma Gandhi and Environmental Movement,” in A.

Raghuramaraju (Ed.), Debating on Gandhi, Oxford University Press, 2006.

R. Guha and M. Gadgil, Ecology and Equity: The Use and Abuse of Nature in Contemporary India, Penguin, 1995.

N. Hanley, J. F. Shogren and B. White, Environmental Economics in Theory and Practice, MacMillan, 2004.

A. Naess and G. Sessions, Basic Principles of Deep Ecology, Ecophilosophy, Vol. 6 (1984).

M. Redclift and G. Woodgate (Eds.), International Handbook of Environmental Sociology, Edward Edgar, 1997.

MA 401 Linear Algebra 3 1 0 8

Vector spaces over fields, subspaces, bases and dimension.

Systems of linear equations, matrices, rank, Gaussian elimination.

Linear transformations, representation of linear transformations by matrices, rank-nullity theorem, duality and transpose.

Determinants, Laplace expansions, cofactors, adjoint, Cramer's Rule.

Eigenvalues and eigenvectors, characteristic polynomials, minimal polynomials, Cayley-Hamilton Theorem, triangulation, diagonal-lization, rational canonical form, Jordan canonical form.

Inner product spaces, Gram-Schmidt ortho-normalization, orthogonal projections, linear functionals and adjoints, Hermitian, self-adjoint, unitary and normal operators, Spectral Theorem for normal operators.

Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness.

Texts / References:

M. Artin, Algebra, Prentice Hall of India, 1994.

K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India), 2003.

S. Lang, Linear Algebra, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1989.

P. Lax, Linear Algebra, John Wiley & Sons, 1997.

H.E. Rose, Linear Algebra, Birkhauser, 2002.

S. Lang, Algebra, 3rd Edition, Springer (India), 2004.

O. Zariski and P. Samuel, Commutative Algebra, Vol. I, Springer, 1975.

MA 403 Real Analysis 3 1 0 8

Review of basic concepts of real numbers: Archimedean property, Completeness.

Metric spaces, compactness, connectedness, (with emphasis on R^n).

Continuity and uniform continuity.

Monotonic functions, Functions of bounded variation; Absolutely continuous functions. Derivatives of functions and Taylor's theorem.

Riemann integral and its properties, characterization of Riemann integrable functions. Improper integrals, Gamma functions.

Sequences and series of functions, uniform convergence and its relation to continuity, differentiation and integration. Fourier series, pointwise convergence, Fejer's theorem, Weierstrass approximation theorem.

Texts / References:

T. Apostol, Mathematical Analysis, 2nd Edition, Narosa, 2002.

K. Ross, Elementary Analysis: The Theory of Calculus, Springer Int. Edition, 2004.

W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1983.

MA 406 General Topology 3 1 0 8

Prerequisites: MA 403 (Real Analysis)

Topological Spaces: open sets, closed sets, neighbourhoods, bases, sub bases, limit points, closures, interiors, continuous functions, homeomorphisms.

Examples of topological spaces: subspace topology, product topology, metric topology, order topology.

Quotient Topology: Construction of cylinder, cone, Moebius band, torus, etc.

Connectedness and Compactness: Connected spaces, Connected subspaces of the real line, Components and local connectedness, Compact spaces, Heine-Borel Theorem, Local -compactness.

Separation Axioms: Hausdorff spaces, Regularity, Complete Regularity, Normality, Urysohn Lemma, Tychonoff embedding and Urysohn Metrization Theorem, Tietze Extension Theorem. Tychnoff Theorem, One-point Compactification.

Complete metric spaces and function spaces, Characterization of compact metric spaces, equicontinuity, Ascoli-Arzela Theorem, Baire Category Theorem. Applications: space filling curve, nowhere differentiable continuous function.

Optional Topics: Topological Groups and orbit spaces, Paracompactness and partition of unity, Stone-Cech Compactification, Nets and filters.

Texts / References

M. A. Armstrong, Basic Topology, Springer (India), 2004.

K. D. Joshi, Introduction to General Topology, New Age International, 2000.
K. D. Joshi, Introduction to General Topology, New Age International, 2000.
J. L. Kelley, General Topology, Van Nostrand, 1955.

J. R. Munkres, Topology, 2nd Edition, Pearson Education (India), 2001.

G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.

MA 408 Measure Theory 3 1 0 8

Prerequisites: MA 403 (Real Analysis)

Semi-algebra, Algebra, Monotone class, Sigma-algebra, Monotone class theorem. Measure spaces.

Outline of extension of measures from algebras to the generated sigma-algebras: Measurable sets; Lebesgue Measure and its properties.

Measurable functions and their properties; Integration and Convergence theorems.

Introduction to L^p-spaces, Riesz-Fischer theorem; Riesz Representation theorem for L^2-spaces. Absolute continuity of measures, Radon-Nikodym theorem. Dual of L^p-spaces.

Product measure spaces, Fubini's theorem.

Fundamental Theorem of Calculus for Lebesgue Integrals (an outline).

Texts / References:

P.R. Halmos, Measure Theory, Graduate Text in Mathematics, Springer-Verlag, 1979.

Inder K. Rana, An Introduction to Measure and Integration (2nd Edition), Narosa Publishing House, New Delhi, 2004.

H.L. Royden, Real Analysis, 3rd Edition, Macmillan, 1988.

MA 410 Multivariable Calculus 2 1 0 6

Prerequisites: MA 403 (Real Analysis), MA 401 (Linear Algebra)
Functions on Euclidean spaces, continuity, differentiability; partial and directional derivatives, Chain Rule, Inverse Function Theorem, Implicit Function Theorem.

Riemann Integral of real-valued functions on Euclidean spaces, measure zero sets, Fubini's Theorem.

Partition of unity, change of variables.

Integration on chains, tensors, differential forms, Poincaré Lemma, singular chains, integration on chains, Stokes' Theorem for integrals of differential forms on chains. (general version). Fundamental theorem of calculus.

Differentiable manifolds (as subspaces of Euclidean spaces), differentiable functions on manifolds, tangent spaces, vector fields, differential forms on manifolds, orientations, integration on manifolds, Stokes' Theorem on manifolds.

Texts / References:

v. Guillemin and A. Pollack, Differential Topology, Prentice-Hall Inc., Englewood Cliffe, New Jersey, 1974.

w. Fleming, Functions of Several Variables, 2nd Edition, Springer-Verlag, 1977.

J.R. Munkres, Analysis on Manifolds, Addison-Wesley, 1991.

W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1984.

M. Spivak, Calculus on Manifolds, A Modern Approach to Classical Theorems of Advanced Calculus, W. A. Benjamin, Inc., 1965.

MA 412 Complex Analysis 3 1 0 8

Complex numbers and the point at infinity. Analytic functions.

Cauchy-Riemann conditions. Mappings by elementary functions. Riemann surfaces. Conformal mappings.

Contour integrals, Cauchy-Goursat Theorem.

Uniform convergence of sequences and series. Taylor and Laurent series. Isolated singularities and residues. Evaluation of real integrals.

Zeroes and poles, Maximum Modulus Principle, Argument Principle, Rouche's theorem.

Texts / References:

J.B. Conway, Functions of One Complex Variable, 2nd Edition, Narosa, New Delhi, 1978.

T.W. Gamelin, Complex Analysis, Springer International Edition, 2001.

R. Remmert, Theory of Complex Functions, Springer Verlag, 1991.

A.R. Shastri, An Introduction to Complex Analysis, Macmilan India, New Delhi, 1999.

MA 414 Algebra I 3 1 0 8

Prerequisite: MA 401 Linear Algebra, MA 419 Basic Algebra

Fields, Characteristic and prime subfields, Field extensions, Finite, algebraic and finitely generated field extensions, Classical ruler and compass constructions, Splitting fields and normal extensions, algebraic closures. Finite fields, Cyclotomic fields, Separable and inseparable extensions.

Galois groups, Fundamental Theorem of Galois Theory, Composite extensions, Examples (including cyclotomic extensions and extensions of finite fields).
Norm, trace and discriminant. Solvability by radicals, Galois' Theorem on solvability. Cyclic extensions, Abelian extensions, Polynomials with Galois groups Sn. Transcendental extensions.

Texts / References:

M. Artin, Algebra, Prentice Hall of India, 1994.

D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Edition, John Wiley, 2002.

J.A. Gallian, Contemporary Abstract Algebra, 4th Edition, Narosa, 1999.

N. Jacobson, Basic Algebra I, 2nd Edition, Hindustan Publishing Co., 1984, W.H. Freeman, 1985.

MA 417 Ordinary Differential Equations 3 1 0 8

Review of solution methods for first order as well as second order equations, Power Series methods with properties of Bessel functions and Legendré polynomials.

Existence and Uniqueness of Initial Value Problems: Picard's and Peano's Theorems, Gronwall's inequality, continuation of solutions and maximal interval of existence, continuous dependence.

Higher Order Linear Equations and linear Systems: fundamental solutions, Wronskian, variation of constants, matrix exponential solution, behaviour of solutions.

Two Dimensional Autonomous Systems and Phase Space Analysis: critical points, proper and improper nodes, spiral points and saddle points.
Asymptotic Behavior: stability (linearized stability and Lyapunov methods).

Boundary Value Problems for Second Order Equations: Green's function, Sturm comparison theorems and oscillations, eigenvalue problems.

Texts / References:

M. Hirsch, S. Smale and R. Deveney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, 2004
M. Hirsch, S. Smale and R. Deveney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, 2004

L. Perko, Differential Equations and Dynamical Systems, Texts in Applied Mathematics, Vol. 7, 2nd Edition, Springer Verlag, New York, 1998.

M. Rama Mohana Rao, Ordinary Differential Equations: Theory and Applications. Affiliated East-West Press Pvt. Ltd., New Delhi, 1980.

D. A. Sanchez, Ordinary Differential Equations and Stability Theory: An Introduction, Dover Publ. Inc., New York, 1968.

MA 419 Basic Algebra 3 1 0 8

Review of basics: Equivalence relations and partitions, Division algorithm for integers, primes, unique factorization, congruences, Chinese Remainder Theorem, Euler ϕ-function.

Permutations, sign of a permutation, inversions, cycles and transpositions. Rudiments of rings and fields, elementary properties, polynomials in one and several variables, divisibility, irreducible polynomials, Division algorithm, Remainder Theorem, Factor Theorem, Rational Zeros Theorem, Relation between the roots and coefficients, Newton's Theorem on symmetric functions, Newton's identities, Fundamental Theorem of Algebra

Rational functions, partial fraction decomposition, unique factorization of polynomials in several variables, Resultants and discriminants.

Groups, subgroups and factor groups, Lagrange's Theorem, homomorphisms, normal subgroups. Quotients of groups, Basic examples of groups: symmetric groups, matrix groups, group of rigid motions of the plane and

finite groups of motions. Cyclic groups, generators and relations, Cayley's Theorem, group actions, Sylow Theorems. Direct products, Structure Theorem for finite abelian groups.

Simple groups and solvable groups, nilpotent groups, simplicity of alternating groups, composition series, Jordan-Holder Theorem. Semidirect products. Free groups, free abelian groups.

Rings, Examples (including polynomial rings, formal power series rings, matrix rings and group rings), ideals, prime and maximal ideals, rings of fractions, Chinese Remainder Theorem for pairwise comaximal ideals. Euclidean Domains, Principal Ideal Domains and Unique Factorizations Domains. Polynomial rings over UFD's.

Texts / References:

M. Artin, Algebra, Prentice Hall of India, 1994.

D. S. Dummit and R. M. Foote, Abstract Algebra, 2nd Edition, John Wiley, 2002.

J. A. Gallian, Contemporary Abstract Algebra, 4th Edition, Narosa, 1999.

K. D. Joshi, Foundations of Discrete Mathematics, Wiley Eastern, 1989.

T. T. Moh, Algebra, World Scientific, 1992.

S. Lang, Undergraduate Algebra, 2nd Edition, Springer, 2001.

S. Lang, Algebra, 3rd Edition, Springer (India), 2004.

J. Stillwell, Elements of Algebra, Springer, 1994.

MA 503 Functional Analysis 3 1 0 8

Prerequisites: MA 401 (Linear Algebra), MA 408 (Measure Theory)

Normed spaces. Continuity of linear maps. Hahn-Banach Extension and Separation Theorems. Banach spaces. Dual spaces and transposes.
Theorems. Banach spaces. Dual spaces and transposes.

Uniform Boundedness Principle and its applications. Closed Graph Theorem, Open Mapping Theorem and their applications. Spectrum of a bounded operator. Examples of compact operators on normed spaces.

Inner product spaces, Hilbert spaces. Orthonormal basis. Projection theorem and Riesz Representation Theorem.

Texts / References:

J.B. Conway, A Course in Functional Analysis, 2nd Edition, Springer, Berlin, 1990.

C. Goffman and G. Pedrick, A First Course in Functional Analysis, Prentice-Hall, 1974.

E. Kreyzig, Introduction to Functional Analysis with Applications, John Wiley & Sons, New York, 1978.

B.V. Limaye, Functional Analysis, 2nd Edition, New Age International, New Delhi, 1996.

A. Taylor and D. Lay, Introduction to Functional Analysis, Wiley, New York, 1980.

MA 504 Operators on Hilbert Spaces 2 1 0 6

Prerequisites: MA 503 (Functional Analysis)

Adjoints of bounded operators on a Hilbert space, Normal, self-adjoint and unitary operators, their spectra and numerical ranges.

Compact operators on Hilbert spaces. Spectral theorem for compact self-adjoint operators.

Application to Sturm-Liouville Problems.

Texts / References:

B.V. Limaye, Functional Analysis, 2nd Edition, New Age International, 1996. J.B.

Conway, A Course in Functional Analysis, 2nd Edition, Springer, 1990.

C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall, 1974.

I. Gohberg and S. Goldberg, Basic Operator Theory, Birkhaüser, 1981.

E. Kreyzig, Introduction to Functional Analysis with Applications, John Wiley & Sons, 1978.

S. G. Mikhlin, Variation Methods in Mathematical Physics, Pergaman Press, Oxford 1964.

J. A. Murdock, Perturbations Theory and Methods, John Wiley and Sons, 1991.

P. D. Miller, Applied asymptotic analysis, American Mathematical Society, 2006.

M. L. Krasnov et.al., Problems and exercises in the calculus of variations, Mir Publishers, 1975.

M. Krasnov et. al., Problems and exercises in integral equations, Mir Publishers, 1971.

MA 510 Introduction to Algebraic Geometry 2 1 0 6

Prerequisites: MA 414 (Algebra 1)

Varieties: Affine and projective varieties, coordinate rings, morphisms and rational maps, local ring of a point, function fields, dimension of a variety.

Curves: Singular points and tangent lines, multiplicities and local rings, intersection multiplicities, Bezout's theorem for plane curves, Max Noether's theorem and some of its applications, group law on a nonsingular cubic, rational parametrization, branches and valuations.

Texts / References:

S.S. Abhyankar, Algebraic Geometry for Scientists and Engineers, American Mathematical Society, 1990.

W. Fulton, Algebraic Curves, Benjamin, 1969.

J. Harris, Algebraic Geometry: A First Course, Springer-Verlag, 1992.

M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press, Cambridge, 1990.

I.R. Shafarevich, Basic Algebraic Geometry, Springer-Verlag, Berlin, 1974.

R.J. Walker, Algebraic Curves, Springer-Verlag, Berlin, 1950.

MA 515 Partial Differential Equations 3 1 0 8

Prerequisites: MA 417 (Ordinary Differential Equations), MA 410 (Multivariable Calculus)

Cauchy Problems for First Order Hyperbolic Equations: method of characteristics, Monge cone.

Classification of Second Order Partial Differential Equations: normal forms and characteristics.

Initial and Boundary Value Problems: Lagrange-Green's identity and uniqueness by energy methods.

Stability theory, energy conservation and dispersion.

Laplace equation: mean value property, weak and strong maximum principle, Green's function, Poisson's formula, Dirichlet's principle, existence of solution using Perron's method (without proof).

Heat equation: initial value problem, fundamental solution, weak and strong maximum principle and uniqueness results.

Wave equation: uniqueness, D'Alembert's method, method of spherical means and Duhamel's principle.

Methods of separation of variables for heat, Laplace and wave equations.

Texts / References:

E. DiBenedetto, Partial Differential Equations, Birkhaüser, 1995.

L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, 1998.

F. John, Partial Differential Equations, 3rd Edition, Narosa, 1979.

E. Zauderer, Partial Differential Equations of Applied Mathematics, 2nd Edition, John Wiley and Sons, 1989.

MA 518 Spectral Approximation 2 1 0 6

Prerequisite: MA 503 Functional Analysis

Spectral decomposition. Spectral sets of finite type. Adjoint and product spaces.

Convergence of operators: norm, collectively compact and ν convergence. Error estimates.

Finite rank approximations based on projections and approximations for integral operators.

A posteriori error estimates.

Matrix formulations for finite rank operators. Iterative refinement of a simple eigenvalue. Numerical examples.

Texts / References:

M. Ahues, A. Largillier and B. V. Limaye, Spectral Computations for Bounded Operators, Chapman and Hall/CRC, 2000.

F. Chatelin, Spectral Approximation of Linear Operators, Academic Press, 1983.

T. Kato, Perturbation Theory of Linear Operators, 2nd Ed., Springer-Verlag, 1980.

MA 521 The Theory of Analytic Functions 2 1 0 6
MA 521 The Theory of Analytic Functions 2 1 0 6

Prerequisites: MA 403 (Real Analysis), MA 412 (Complex Analysis)

Maximum Modulus Theorem. Schwarz Lemma. Phragmen-Lindelof Theorem.

Riemann Mapping Theorem. Weierstrass Factorization Theorem.

Runge's Theorem. Simple connectedness. Mittag-Leffler Theorem.

Schwarz Reflection Principle.

Basic properties of harmonic functions.

Picard Theorems.

Texts / References:

L. Ahlfors, Complex Analysis, 3rd Edition, McGraw-Hill, 1979.

J.B. Conway, Functions of One Complex Variable, 2nd Edition, Narosa, 1978.

T.W. Gamelin, Complex Analysis, Springer International, 2001.

R. Narasimhan, Theory of Functions of One Complex Variable, Springer (India), 2001.

W. Rudin, Real and Complex Analysis, 3rd Edition, Tata McGraw-Hill, 1987.

MA 523 Basic Number Theory 2 1 0 6

Prerequisites: MA 419 (Basic Algebra)

Infinitude of primes, discussion of the Prime Number Theorem, infinitude of primes in specific arithmetic progressions, Dirichlet's theorem (without proof).

Arithmetic functions, Mobius inversion formula. Structure of units modulo *n*, Euler's phi function

Congruences, theorems of Fermat and Euler, Wilson's theorem, linear congruences, quadratic residues, law of quadratic reciprocity.

Binary quadratics forms, equivalence, reduction, Fermat's two square theorem, Lagrange's four square theorem.

Continued fractions, rational approximations, Liouville's theorem, discussion of Roth's theorem, transcendental numbers, transcendence of e and π.

Diophantine equations: Brahmagupta's equation (also known as Pell's equation), the The equation, Fermat's method of descent, discussion of the Mordell equation.

Texts / References

W.W. Adams and L.J. Goldstein, Introduction to the Theory of Numbers, 3rd Edition, Wiley Eastern, 1972.

A. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press, 1984.

I. Niven and H.S. Zuckerman, An Introduction to the Theory of Numbers, 4th Edition, Wiley, 1980.

MA 524 Algebraic Number Theory 2 1 0 6

Prerequisites: MA 414 Algebra I (Exposure)

Algebraic number fields. Localisation, discrete valuation rings.

Integral ring extensions, Dedekind domains, unique factorisation of ideals. Action of the Galois group on prime ideals.

Valuations and completions of number fields, discussion of Ostrowski's theorem, Hensel's lemma, unramified, totally ramified and tamely ramified extensions of p-adic fields.

Discriminants and Ramification. Cyclotomic fields, Gauss sums, quadratic reciprocity revisited.

The ideal class group, finiteness of the ideal class group, Dirichlet units theorem.
The ideal class group, finiteness of the ideal class group, Dirichlet units theorem.

Texts / References

K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, 2nd Edition, Springer-Verlag, Berlin, 1990.

S. Lang, Algebraic Number Theory, Addison-Wesley, 1970.

D. A. Marcus, Number Fields, Springer-Verlag, 1977.

MA 525 Dynamical Systems 2 1 0 6

Prerequisites: MA 417 (Ordinary Differential Equations)

Linear Systems: Review of stability for linear systems of two equations.

Local Theory for Nonlinear Planar Systems: Flow defined by a differential equation, Linearization and stable manifold theorem, Hartman-Grobman theorem, Stability and Lyapunov functions, Saddles, nodes, foci, centers and nonhyperbolic critical points. Gradient and Hamiltonian systems.

Global Theory for Nonlinear Planar Systems: Limit sets and attractors, Poincaré map, Poincaré Benedixson theory and Poincare index theorem.

Bifurcation Theory for Nonlinear Systems: Structural stability and Peixoto's theorem, Bifurcations at nonhyperbolic equilibrium points.

Texts / References:

L. Perko, Differential Equations and Dynamical Systems, Springer Verlag, 1991.

M. W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and Linear Algebra, Academic Press, 174.

P. Hartman, Ordinary Differential Equations, 2nd edition, SIAM 2002.
C. Chicone, Ordinary Differential Equations with Applications, 2nd Edition, Springer, 2006.

MA 526 Commutative Algebra 2 1 0 6

Prerequisites: MA 505 (Algebra II)

Dimension theory of affine algebras: Principal ideal theorem, Noether normalization lemma, dimension and transcendence degree, catenary property of affine rings, dimension and degree of the Hilbert polynomial of a graded ring, Nagata's altitude formula, Hilbert's Nullstellensatz, finiteness of integral closure.

Associated primes of modules, degree of the Hilbert polynomial of a graded module, Hilbert series and dimension, Dimension theorem, Hilbert-Samuel multiplicity, associativity formula for multiplicity,

Complete local rings: Basics of completions, Artin-Rees lemma, associated graded rings of filtrations, completions of modules, regular local rings

Basic Homological algebra: Categories and functors, derived functors, Hom and tensor products, long exact sequence of homology modules, free resolutions, Tor and Ext, Koszul complexes.

Cohen-Macaulay rings: Regular sequences, quasi- regular sequences, Ext and depth, grade of a module, Ischebeck's theorem, Basic properties of Cohen-Macaulay rings, Macaulay's unmixed theorem, Hilbert-Samuel multiplicity and Cohen-Macaulay rings, rings of invariants of finite groups.

Optional Topics: Face rings of simplicial complexes, shellable simplicial complexes and their face rings. Dedekind Domains and Valuation Theory.

Texts / References:

D. Eisenbud, Commutative Algebra (with a view toward algebraic geometry), Graduate Texts in Mathematics 150, Springer-Verlag, 2003.
Texts in Mathematics 150, Springer-Verlag, 2003.

H. Matsumura, Commutative ring theory, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press, 1980.

X. Bruns and J. Herzog, Cohen-Macaulay Rings, Revised edition, Cambridge Studies in Advanced Mathematics No. 39, Cambridge University Press, 1998.

MA 528 Hyperplane Arrangements 3 0 0 6

Faces, flats, cones, lunes. Distance functions Birkhoff monoid, Tits monoid & Janus monoid Lie and Zie elements Eulerian idempotents, Dynkin idempotents, Joyal-Klyachko-Stanley theorem Orlik-Solomon algebra Incidence algebras and operads.

Texts / References:
P. Orlik and H. Terao, Arrangements of hyperplanes, Springer, 1992.

C. De Concini and C. Procesi,.Topics in hyperplane arrangements, polytopes and box-splines. Springer, 2011.

A. Dimca Hyperplane arrangements. An introduction. Springer, 2017.

M. Aguiar and S. Mahajan. Topics in hyperplane arrangements. AMS, 2017.

MA 530 Nonlinear Analysis 2 1 0 6

Prerequisites: MA 503 (Functional Analysis)

Fixed Point Theorems with Applications: Banach contraction mapping theorem, Brouwer fixed point theorem, Leray-Schauder fixed point theorem.

Calculus in Banach spaces: Gateaux as well as Frechet derivatives, chain rule, Taylor's expansions, Implicit function theorem with applications, subdifferential.

Monotone Operators: maximal monotone operators with properties, surjectivity theorem with applications.

Degree theory and condensing operators with applications.

Texts / References:

M.C. Joshi and R.K. Bose, Some Topics in Nonlinear Functional Analysis, Wiley Eastern Ltd., New Delhi, 1985.

E. Zeilder, Nonlinear Functional Analysis and Its Applications, Vol. I (Fixed Point Theory), Springer Verlag, Berlin, 1985.

MA 532 Analytic Number Theory 2 1 0 6

Prerequisites: MA 414 (Algebra I), MA 412 (Complex Analysis)

The Wiener-Ikehara Tauberian theorem, the Prime Number Theorem. Dirichlet's theorem for primes in an Arithmetic Progression.

Zero free regions for the Riemann-zeta function and other L-functions.

Euler products and the functional equations for the Riemann zeta function and Dirichlet L-functions.

Modular forms for the full modular group, Eisenstein series, cusp forms, structure of the ring of modular forms.

Hecke operators and Euler product for modular forms.

The L-function of a modular form, functional equations. Modular forms and the sums of four squares.

Optional topics: Discussion of L-functions of number fields and the Chebotarev Density Theorem. Phragmen-Lindelof Principle, Mellin inversion formula, Hamburger's theorem. Discussion of Modular forms for congruence subgroups. Discussion of Artin's holomorphy conjecture and higher reciprocity laws. Discussion of elliptic curves and the Shimura-Taniyama conjecture (Wiles' Theorem)

Texts / References:

S. Lang, Algebraic Number Theory, Addison-Wesley, 1970.

J.P. Serre, A Course in Arithmetic, Springer-Verlag, 1973.

T. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1976.

MA 533 Advanced Probability Theory 2 1 0 6

Probability measure, probability space, construction of Lebesgue measure, extension theorems, limit of events, Borel-Cantelli lemma.
Random variables, Random vectors, distributions, multidimensional distributions, independence.

Expectation, change of variable theorem, convergence theorems.

Sequence of random variables, modes of convergence. Moment generating function and characteristics functions, inversion and uniqueness theorems, continuity theorems, Weak and strong laws of large number, central limit theorem.

Radon Nikodym theorem, definition and properties of conditional expectation, conditional distributions and expectations.

Texts / References:

P. Billingsley, Probability and Measure, 3rd Edition, John Wiley & Sons, New York, 1995.

J. Rosenthal, A First Look at Rigorous Probability, World Scientific, Singapore, 2000.

A.N. Shiryayev, Probability, 2nd Edition, Springer, New York, 1995.

K.L. Chung, A Course in Probability Theory, Academic Press, New York, 1974.

MA 534 Modern Theory of Partial Differential Equations 2 1 0 6

Prerequisites: MA 503 (Functional Analysis), MA 515 (Partial Differential Equations)

Theory of distributions: supports, test functions, regular and singular distributions, generalised derivatives.

Sobolev Spaces: definition and basic properties, approximation by smooth functions, dual spaces, trace and imbedding results (without proof).

Elliptic Boundary Value Problems: abstract variational problems, Lax-Milgram Lemma, weak solutions and wellposedness with examples, regularity result, maximum principles, eigenvalue problems.

Semigroup Theory and Applications: exponential map, C_0-semigroups, Hille-Yosida and Lummer-Phillips theorems, applications to heat and wave equations.

Texts / References:

S. Kesavan, Topics in Functional Analysis Wiley Eastern Ltd., New Delhi, 1989.

M. Renardy and R.C. Rogers, An Introduction to Partial Differential Equations,2nd Edition, Springer Verlag International Edition, New York, 2004.

L.C. Evans, Partial Differential Equations, American Mathematical Society, Providence, 1998.

MA 538 Representation Theory of Finite Groups 2 1 0 6

Prerequisite : MA 414 (Algebra I)

Representations, Subrepresentations, Tensor products, Symmetric and Alternating Squares.

Characters, Schur's lemma, Orthogonality relations, Decomposition of regular representation, Number of irreducible representations, canonical decomposition and explicit decompositions. Subgroups, Product groups, Abelian groups. Induced representations.

Examples: Cyclic groups, alternating and symmetric groups.

Integrality properties of characters, Burnside's p^aq^b theorem. The character of induced representation, Frobenius Reciprocity Theorem, Meckey's irreducibility criterion, Examples of induced representations, Representations of supersolvable groups.

Texts / References:

Burrow, Representation Theory of Finite Groups, Academic Press, 1965.

M. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, 1983.

S. Lang, Algebra, 3rd Edition, Springer (India), 2004.

J.-P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.

MA 539 Spline Theory and Variational Methods 2 1 0 6

Even Degree and Odd Degree Spline Interpolation, end conditions, error analysis and order of convergence. Hermite interpolation, periodic spline interpolation. B-Splines, recurrence relation for B-splines, curve fitting using splines, optimal quadrature.

Tensor product splines, surface fitting, orthogonal spline collocation methods.

Texts / References

C. De Boor, A Practical Guide to Splines, Springer-Verlag, Berlin, 1978.

H.N. Mhaskar and D.V. Pai, Fundamentals of Approximation Theory, Narosa Publishing House, New Delhi, 2000.

P.M. Prenter, Splines and Variational Methods, Wiley-Interscience, 1989.

MA 540 Numerical Methods for Partial Differential Equations 2 1 0 6

Prerequisites: MA 515 (Partial Differential Equations), SI 517 (Numerical Analysis)

Finite differences: Grids, Finite-difference approximations to derivatives.

Linear Transport Equation: Upwind, Lax-Wendroff and Lax -Friedrich schemes, von-Neumann stability analysis, CFL condition, Lax-Richtmyer equivalence theorem, Modified equations, Dissipation and dispersion.

Heat Equation: Initial and boundary value problems (Dirichlet and Neumann), Explicit and implicit methods (Backward Euler and Crank-Nicolson schemes) with consistency and stability, Discrete maximum principle, ADI methods for two dimensional heat equation (including LOD algorithm).

Poisson's Equation: Finite difference scheme for initial and boundary value problems, Discrete maximum principle, Iterative methods for linear systems (Jacobi, Gauss-Seidel, SOR methods and Conjugate Gradient method), Peaceman-Rachford algorithm (ADI) for linear systems.

Wave Equation: Explicit schemes and their stability analysis, Implementation of boundary conditions.

Lab Component: Exposure to MATLAB and computational experiments based on the algorithms discussed in the course.

Texts / References:

K. W. Morton and D. Mayers, Numerical Solution for Partial Differential Equations, 2nd edition, Cambridge, 2005.

G. D. Smith, Numerical Solutions of Partial Differential Equations, 3rd Edition, Calrendorn Press, 1985.

J. C. Strikwerda, Finite difference Schemes and Partial Differential Equations, Wadsworth and Brooks/ Cole, 1989.

J. W. Thomas, Numerical Partial Differential Equations : Finite Difference Methods, Texts in Applied Mathematics, Vol. 22, Springer Verlag, 1999.

J. W. Thomas, Numerical Partial Differential Equations: Conservation Laws and Elliptic Equations, Texts in Applied Mathematics, Vol. 33, Springer Verlag, 1999.

R. Mitchell and S. D. F. Griffiths, The Finite Difference Methods in Partial Differential Equations, Wiley and Sons, NY, 1980.

MA 556 Differential Geometry 2 1 0 6

Prerequiste: MA 410 Multivariable Calculus

Graphs and level sets of functions on Euclidean spaces, vector fields, integral curves of vector fields, tangent spaces.

Surfaces in Euclidean spaces, vector fields on surfaces, orientation, Gauss map. Geodesics, parallel transport, Weingarten map.

Curvature of plane curves, arc length and line integrals, Curvature of surfaces. Parametrized surfaces, local equivalence of surfaces.

Gauss-Bonnet Theorem, Poincare-Hopf Index Theorem.

Texts / References

M. doCarmo, Differential Geometry of Curves and Surfaces, Prentice Hall, 1976.

B. O'Neill, Elementary Differential Geometry, Academic Press, 1966.

J.J. Stoker, Differential Geometry, Wiley-Interscience, 1969.

J. A. Thorpe, Elementary Topics in Differential Geometry, Springer (India), 2004.

MA 562 The Mathematical Theory of Finite Elements 2 1 0 6

Prerequisites: MA 515 (Partial Differential Equations), MA 503 (Functional Analysis)

Sobolev Spaces: basic elements, Poincare inequality. Abstract variational formulation and elliptic boundary value problem. Galerkin formulation and Cea's Lemma. Construction of finite element spaces. Polynomial approximations and interpolation errors.

Convergence analysis: Aubin-Nitsche duality argument; non-conforming elements and numerical integration; computation of finite element solutions.

Parabolic initial and boundary value problems: semidiscrete and completely discrete schemes with convergence analysis.

Lab component: Implementation of algorithms and computational experiments using MATLAB.

Texts / References:

K. E. Brenner and R. Scott, The Mathematical Theory of Finite Element Methods, Springer- Verlag, 1994.

P.G. Ciarlet, The Finite Element Methods for Elliptic Problems, North Holland, 1978.

C. Johnson, Numerical solutions of Partial Differential Equations by Finite Element Methods, Cambridge University Press, 1987.

C. Mercier, Lectures on Topics in Finite Element Solution of Elliptic Problems, TIFR Lectures on Mathematics and Physics Vol. 63, Narosa, 1979.

MA 581 Elements of Differential Topology 2 1 0 6

Prerequisites: MA 410 (Multivariable Calculus)

Differentiable Manifolds in Rn: Review of inverse and implicit function theorems; tangent spaces and tangent maps; immersions; submersions and embeddings. Regular Values: Regular and critical values; regular inverse image theorem; Sard's theorem; Morse lemma. Transversality: Orientations of manifolds; oriented and mod 2 intersection numbers; degree of maps. Application to the Fundamental Theorem of Algebra

*Lefschetz theory of vector fields and flows: Poincare-Hopf index theorem; Gauss-Bonnet theorem.

*Abstract manifolds: Examples such as real and complex projective spaces and Grassmannian varieties; Whitney embedding theorems.

(*indicates expository treatment intended for these parts of the syllabus.)

Texts / References

B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov, Modern Geometry – Methods and Applications, Part II: The Geometry and Topology of Manifolds, Springer-Verlag, 1985.

V. Guillemin and A Pollack, Differential Topology Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1974.

J. Milnor, Topology from the Differential View-point, University Press of Virginia, Charlottsville 1990.

A. R. Shastri, Elements of Differential Topology, CRC Press, 2011.

MA 593 Optional Project Stage I

MA 598 Optional Project Stage II

SI 416 Optimization 2 0 2 6

Unconstrained optimization using calculus (Taylor's theorem, convex functions, coercive functions).

Unconstrained optimization via iterative methods (Newton's method, Gradient/ conjugate gradient based methods, Quasi-Newton methods).

Constrained optimization (Penalty methods, Lagrange multipliers, Kuhn-Tucker conditions. Linear programming (Simplex method, Dual simplex, Duality theory). Modeling for Optimization.

Text / References:
M.Bazarra and C.Shetty, Nonlinear Programming, Theory and Algorithms, Wiley,1979. E. M. L. Beale, Introduction to Optimization, John Wiley, 1988.

M.C. Joshi and K. Moudgalya, Optimization: Theory and Practice, Narosa, 2004.

SI 507 Numerical Analysis 3 0 2 8

Prerequisites: Exposure to a first course in Calculus

Errors: Floating-point approximation of a number, Loss of significance and error propagation, Stability in numerical computation.

Linear Systems: Gaussian elimination with pivoting strategy, LU factorization, Residual corrector method, Solution by iteration (Jacobi and Gauss -Seidal with convergence analysis), Matrix norms and error in approximate solution, Eigenvalue problem (Power method), Gershgorin's theorem (without proof).
Nonlinear Equations: Bisection method, Fixed-point iteration method, Secant method, Newton's method, Rate of convergences, Solution of a system of nonlinear equations, Unconstrained optimization.

Interpolation by Polynomials: Lagrange interpolation, Newton interpolation and divided differences, Error of the interpolating polynomials, Piecewise linear and cubic spline interpolation, Trigonometric interpolation, Data fitting and least-squares approximation problem.

Differentiation and Integration: Difference formulae, Some basic rules of integration, Adaptive quadratures, Gaussian rules, Composite rules, Error formulas.

Differential Equations: Euler method, Runge-Kutta methods, Multi -step methods, Predictor-Corrector methods Stability and convergence, Two point boundary value problems.

Lab Component: Exposure to MATLAB implementation.

Texts / References:
K. E. Atkinson, An Introduction to Numerical Analysis, 2nd Edition, Wiley-India, 1989.

S. D. Conte and C. de Boor, Elementary Numerical Analysis - An Algorithmic Approach, 3rd Edition, McGraw-Hill, 1981.

R. L. Burden and J. D. Faires, Numerical Analysis, 7th Edition, Thomson, 2001.

SI 419 Combinatorics 2 1 0 6

Prerequisites: MA 401 (Linear Algebra)

Basic Combinatorial Objects: Sets, multisets, partitions of sets, partitions of numbers, finite vector spaces, permutations, graphs etc.

Basic Counting Coefficients: The twelve fold way, binomial, q-binomial and the Stirling coefficients, permutation statistics, etc.

Sieve Methods: Principle of inclusion exclusion, permutations with restricted positions, Sign-reversing involutions, determinants etc
.
Introduction to combinatorial reciprocity. Introduction to symmetric functions.

Texts/ References:

C. Berge, Principles of Combinatorics, Academic Press, 1972.

K. D. Joshi, Foundations of Discrete Mathematics, Wiley Eastern, 2000.

R. P. Stanley, Enumerative Combinatorics, Vol. I, Wadsworth and Brooks/Cole, 1986.

SI 527 Introduction to Derivatives Pricing 2 1 0 6

Prerequisites: SI 417 (Introduction to Probability Theory) or MA 533 (Advanced Probability Theory)
Basic notions – Cash flow, present value of a cash flow, securities, fixed income securities, types of markets.

Forward and futures contracts, options, properties of stock option prices, trading strategies involving options, option pricing using Binomial trees, Black – Scholes model, Black – Scholes formula, Risk-Neutral measure, Delta – hedging, options on stock indices, currency options.

Texts / References

D. G. Luenberger, Investment Science, Oxford University Press, 1998.

J. C. Hull, Options, Futures and Other Derivatives, 4th Edition, Prentice-Hall, 2000.

J. C. Cox and M. Rubinstein, Options Market, Englewood Cliffs, N.J.: Prentice Hall, 1985.

C. P Jones, Investments, Analysis and Measurement, 5th Edition, John Wiley and Sons, 1996.

NEW COURSES

MA 5101 Algebra II 2 1 0 6

Prerequisites: MA 414 Algebra I

Modules, submodules, quotient modules and module homomorphisms.

Generation of modules, direct sums and free modules. Tensor products of modules. Exact sequences. Hom and Tensor duality.

Finitely generated modules over principal ideal domains, invariant factors, elementary divisors, rational canonical forms. Applications to finitely generated abelian groups and linear transformations.

Noetherian rings and modules, Hilbert basis theorem, Primary decomposition of ideals in Noetherian rings. Integral extensions, Going-up and Going-down theorems, Extension and contraction of prime ideals, Noether's Normalization Lemma, Hilbert's Nullstellensatz.

Texts / References:

M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison Wesley, 1969.

D. S. Dummit and R. M. Foote, Abstract Algebra, 2nd Edition, John Wiley, 2002.

N. Jacobson, Basic Algebra I and II, 2nd Edition, W. H. Freeman, 1985 and 1989.

S. Lang, Algebra, 3rd Edition, Springer (India), 2004.

O. Zariski and P. Samuel, Commutative Algebra, Vol. I, Springer, 1975.

MA 5102 Basic Algebraic Topology 2 1 0 6

Prerequisites: MA 406 (General Topology)

Paths and homotopy, homotopy equivalence, contractibility, deformation retracts.

Basic constructions: cones, mapping cones, mapping cylinders, suspension.

Fundamental groups. Examples (including the fundamental group of the circle) and applications (including Fundamental Theorem of Algebra, Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem, both in dimension two). Van Kampen's Theorem, Covering spaces, lifting properties, deck transformations. universal coverings (existence theorem optional).

Singular Homology. Mayer-Vietoris Sequences. Long exact sequence of pairs and triples. Homotopy invariance and excision theorem (without proof).
Applications of homology: Jordan-Brouwer separation theorem, invariance of dimension, Hopf's Theorem for commutative division

algebras with identity, Borsuk-Ulam Theorem, Lefschetz Fixed Point Theorem.

Texts / References:

M. J. Greenberg and J. R. Harper, Algebraic Topology, Benjamin, 1981.

W. Fulton, Algebraic topology: A First Course, Springer-Verlag, 1995.

A. Hatcher, Algebraic Topology, Cambridge Univ. Press, Cambridge, 2002.

W. Massey, A Basic Course in Algebraic Topology, Springer-Verlag, Berlin, 1991.

J. R. Munkres, Elements of Algebraic Topology, Addison-Wesley, 1984.

J. J. Rotman, An Introduction to Algebraic Topology, Springer (India), 2004.

H. Seifert and W. Threlfall, A Textbook of Topology, translated by M. A. Goldman, Academic Press, 1980.

J. W. Vick, Homology Theory: An Introduction to Algebraic Topology, 2nd Edition, Springer-Verlag, 1994.

MA 5103 Algebraic Combinatorics 2 1 0 6

Prerequisites: MA 401 (Linear Algebra)

A selection of topics from the following:

Algebraic Graph theory: adjacency and Laplacian matrices of a graph, Matrix-Tree theorem, Cycle space and Bond space.

Algebraic Sperner theory: Sperner property of posets, algebraic characterization of strong Sperner property, unimodality of q-binomial cofficients.

Young Tableaux: Up-Down operators on the Young lattice and counting tableaux, RSK correspondence.

Enumeration under group action: Burnside's lemma, Polya theory.

Spectral Graph theory: Isoperimetric problems, Flows and Cheeger constants, Quasirandomness, expanders, and eigenvalues, random walks on graphs.

The Combinatorial Nullstellensatz and some of its applications.

Linear Algebra methods in Combinatorics.

Association Schemes.

Electrical Networks and resistances. Connections to Graph sparsification.

Texts / References:

N. Alon, Combinatorial Nullstellensatz, Combinatorics, Probability, and Computing, Vol. 8 (1999), pp. 7-29.

R. P. Stanley, Algebraic Combinatorics: Walks, Trees, Tableaux, and More, Springer, 2013.

C. Godsil and G. F. Royle, Algebraic Graph Theory, Springer, 2001.

F. Chung, Spectral Graph Theory, CBMS Regional Conference Series in Math., No. 92, American Mathematical Society, 1991.

L. Babai and P. Frankl, Linear Algebra Methods in Combinatorics, Department of Computer Science, University of Chicago, Preliminary version, 1992.

MA 5104 Hyperbolic Conservation Laws 2 1 0 6

Prerequisites: MA 515 Partial Differential Equations (Exposure)

Basic Concepts: Definition and examples, Loss of regularity, Weak solution, Rankine-Hugoniot jump condition, Entropy solution.

Scalar Conservation Laws: Existence of an entropy solution, Uniqueness of the entropy solution, Asymptotic behavior of the entropy solution, The Riemann problem.

System of Conservation Laws: Linear hyperbolic system with constant coefficients,

Nonlinear case, Simple waves and Riemann invariants, Shock waves and contact discontinuities, Characteristic curves and entropy conditions, Solution of the Riemann problem, The Riemann problem for the p-system.

Texts / References:

L. C. Evans, Partial Differential Equations, American Mathematical Society, 2010.

E. Godlewski and P.-A. Raviart, Numerical Approximation of Hyperbolic Systems of Conservation Laws, Springer, 1996.

A. Bressan, Hyperbolic Systems of Conservation Laws – The One-Dimensional Cauchy Problem, Oxford University Press, 2000.

J. Smoller, Shock Waves and Reaction-Diffusion Equations, Springer, 1994.

MA 5105 Coding Theory 2 1 0 6

Prerequisites: MA 401 (Linear Algebra)

Basic Concepts: Idea behind use of codes, block codes and linear codes, repetition codes, nearest neighbour decoding, syndrome decoding, requisite basic ideas in probability, Shannon's theorem (without proof).

Good linear and nonlinear codes: Binary Hamming codes, dual of a code, constructing codes by various operations, simplex codes, Hadamard matrices and codes constructed from Hadamard and conference matrices, Plotkin bound and various other bounds, Gilbert-Varshamov bound.

Reed-Muller and related codes: First order Reed-Muller codes, RM code of order r, Decoding and Encoding using the algebra of finite field with characteristic two.

Perfect codes: Weight enumerators, Kratchouwk polynomials, Lloyd's theorem, Binary and ternary Golay codes, connections with Steiner systems.

Cyclic codes: The generator and the check polynomial, zeros of a cyclic code, the idempotent generators, BCH codes, Reed-Solomon codes, Quadratic residue codes, generalized RM codes.

Optional topics;

Codes over Z_4 : Quaternary codes over Z_4, binary codes derived from such codes, Galois rings over Z_4, cyclic codes over Z_4.

Goppa codes: the minimum distance of Goppa codes, generalized BCH codes, decoding of Goppa codes and their asymptotic behaviour.

Algebraic geometry codes: algebraic curves and codes derived from them, Riemann-Roch theorem (statement only) and applications to algebraic geometry codes.

Texts / References :

J. H. van Lint, Introduction to Coding Theory, Springer, 1999.

W. C. Huffman and V. Pless, Fundamentals of Error Correcting Codes, Cambridge University Press, 2003.

J. MacWilliams and N. J. A. Sloane, The Theory of Error Correcting Codes, North-Holland, 1977.

S. Ling and C. Xing, Coding Theory: A First Course, Cambridge University Press, 2004.

MA 5106 Introduction to Fourier Analysis 2 1 0 6

Prerequisites: MA 403 Real Analysis

Basic Properties of Fourier Series: Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Summability, Fejer's theorem, Poisson Kernel and Dirichlet problem in the unit disc. Mean square Convergence, Example of Continuous functions with divergent Fourier series.

Distributions and Fourier Transforms: Calculus of Distributions, Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians.
Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs (Laplace, Heat and Wave Equations), Schrodinger-Equation and Uncertainty principle.

Paley-Wienner Theorems, Poisson Summation Formula: Radial Fourier transforms and Bessel's functions. Hermite functions.

Texts / References:

R. S. Strichartz, A Guide to Distributions and Fourier Transforms, CRC Press, 1994.

E. M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, 2003.

I. Richards and H. Youn, Theory of Distributions: A Nontechnical Introduction, Cambridge University Press, 1990.

MA 5107 Continuum Mechanics 2 1 0 6

Prerequisites: Exposure to MA 417 (Ordinary Differential Equations) and MA 410 (Multivariable Calculus)

Preliminaries: Tensor algebra and calculus, Continuum mass and force concepts.

Kinematics of Continuous Media: Deformation, Changes in distance, angles, volume, area, Particle derivatives, Measures of strain: Cauchy-Green strain tensor.

Balance Laws of motion: Lagrangean and Eulerian forms of Conservation laws for mass, linear and angular momentum, and energy, Frame-indifference.

Constitutive relations: Constitutive laws for solids and fluids, principle of material frame indifference, discussion of isotropy, linearized elasticity, fluid mechanics.
indifference, discussion of isotropy, linearized elasticity, fluid mechanics.

Texts / References:

O. Gonzalez and A. M. Stuart, A First Course in Continuum mechanics, Cambridge University Press, 2008.

M. Gurtin, An Introduction to Continuum Mechanics, Academic press, 1981.

J. N. Reddy, An Introduction to Continuum Mechanics with Applications, Cambridge University Press, 2008.

J. N. Reddy, Principles of Continuum Mechanics: A Study of Conservation Principles with Applications, Cambridge University Press, 2010.

Y. R. Talpaert, Tensor analysis and Continuum Mechanics, Springer, 2003.

R. Temam and A. Miranville, Mathematical Modelling in Continuum Mechanics, Cambridge University Press, 2005.

MA 5108 Lie Groups and Lie Algebras 2 1 0 6

Prerequisites: Exposure to MA 401 (Linear Algebra) and MA 403 (Real Analysis)

Introduction, Examples: Rotations of the plane, Quaternions and space rotations, SU(2) and SO(3), The Cartan-Dieudonné Theorem, Quaternions and rotations in R4, SU(2)xSU(2) and SO(4). Matrix Lie groups: definitions and examples. The symplectic, orthogonal and unitary groups, connectedness, compactness. Maximal tori. centres and discrete subgroups The exponential map, Lie algebras The matrix exponential, tangent spaces, the Lie algebra of a Lie group. Complexification, the matrix logarithm, the exponential map, One parameter subgroups, the functor from Lie groups to Lie algebras The adjoint mapping,

normal subgroups and Lie algebras The Campbell-Baker-Hausdorff Theorem, simple connectivity, simply connected Lie groups and their characterization by Lie algebras, covering groups.

Texts / References:

J.Stillwell, Naive Lie Theory, Springer, 2008.

A. Kirillov Jr., Introduction to Lie Groups and Lie Algebras, Cambridge University Press, 2008.

MA 5109 Graph Theory 2 1 0 6

Basic Concepts: various kinds of graphs, simple graphs, complete graph, walk, tour, path and cycle, Eulerian graph, bipartite graph (characterization), Havel-Hakimi theorem and Erdos -Gallai theorem (statement only), hypercube graph, Petersen graph, trees, forests and spanning subgraphs, distances, radius, diameter, center of a graph, the number of distinct spanning trees in a complete graph.

Trees: Kruskal and Prim algorithms with proofs of correctness, Dijkstra's a algorithm, Breadth first and Depth first search trees, rooted and binary trees, Huffman's algorithm

Matchings: augmenting path, Hall's matching theorem, vertex and edge cover, independence number and their connections, Tutte's theorem for the existence of a 1-factor in a graph

Connectivity k-vertex and edge connectivity, blocks, characterizations of 2- connected graphs, Menger's theorem and applications, Network flows, Ford- Fulkerson algorithm, Supply-demand theorem and the Gale-Ryser theorem on degree sequences of bipartite graphs

Graph Colourings chromatic number, Greedy algorithm, bounds on chromatic numbers, interval graphs and chordal graphs (with simplicial elimination ordering), Brook's theorem and graphs with no triangles but large chromatic number, chromatic polynomials.

Hamilton property Necessary conditions, Theorems of Dirac and Ore, Chvatal's theorem and toughness of a graph, Non-Hamiltonian graphs with large vertex degrees.

Planar graphs Embedding a graph on plane, Euler's formula, non-planarity of K_5 and $K_{3,3}$, classification of regular polytopes, Kuratowski's theorem (no proof), 5-colour theorem.

Ramsey theory Bounds on $R(p, q)$, Bounds on $R_k(3)$: colouring with k colours and with no monochromatic K_3, application to Schur's theorem, Erdos and Szekeres theorem on points in general position avoiding a convex m-gon.

Texts / References:

D. B. West, Introduction to Graph Theory, Prentice Hall of India, 2001.

J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Springer-Verlag, 2008.

R. Diestel, Introduction to Graph Theory, Springer-Verlag, 2010.

MA5110 Noncommutative Algebra 2 1 0 6

Prerequisites: MA 419 (Basic Algebra)

Wedderburn-Artin Theory: semisimple rings and modules, Weddereburn and Artin's structure theorem of semisimple rings.

Jacobson radical theory: Jacobson radical, Jacobson semisimple rings (or semiprimitive rings), nilpotent ideal, Hopkins and Levitzki theorem, Jacobson radical under base change, semisimplicity of group rings.

Prime and primitive rings: prime and semiprime ideal (and ring), primitive ring and ideal, Jacobson-Chevalley's density theorem, Structure theorem for left primitive rings, Jacobson-Herstein's commutativity theorem.

Introduction to division rings: Wedderburn's (little) theorem, algebraic division algebras over reals (Frobenius theorem), construction of division algebras, polynomials over division rings.

Ordered structures in rings: orderings and preorderings in rings,formally real ring, ordered division rings.

Local rings, semilocal rings and idempotents: Krull-Schmidt-Azumaya theorem on uniqueness of indecomposable summands of a module, stable range of a ring and cancellation of modules.

Brauer group and Clifford algebras.

Text/References:

N. Jacobson, Basic Algebra, Vol. I and II, Dover Publications, 2009.

S. Lang, Algebra, 3rd Edition, Springer Verlag, 2002

T. Y. Lam, A First Course in Noncommutative Rings, 2nd edition, Springer, 2001.

A. Knapp, Advanced Algebra, Birkhauser, 2007.

MA 5111 - Theory of Finite Semigroups
3 0 0 6

Monoids and their linearized algebras Bands, left regular bands Hyperplane arrangements: Birkhoff monoid, Tits monoid & Janus monoid Idempotents and simple modules Quivers of band algebras Noncommutative zeta and Mobius functions Karoubi envelopes of semigroups.

Texts / References:

P. Grillet. Semigroups. an introduction to the structure theory, Marcel Dekker Inc., 1995

J. Rhodes and B. Steinberg, The q-theory of finite semigroups. Springer, 2009

B. Steinberg.Representation theory of finite monoids. Springer, 2016

M. Aguiar and S. Mahajan. Topics in hyperplane arrangements. AMS, 2017

MA 5112 Introduction to Mathematical Methods 2 1 0 6

Prerequisites: MA 515 (Partial Differential Equations)

Asympotic expansions:
Watson's lemma, method of stationary phase and saddle point method. Applications to differential equations. Behaviour of solutions near an irregular singular point, Stoke's phenomenon. Method of strained coordinates and matched asymptotic expansions, Lindstedt expansions.

Calculus of variations: Classical methods.

Integral equations: Volterra integral equations of first and second kind. Iterative methods and Neumann series.

Texts / References:

C. M. Bender and S. A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill Book Co., 1978.

R. Courant & D. Hilbert, Methods of Mathematical Physics, Vol. I & II, Wiley Eastern, 1975.

J. Kevorkian and J.D. Cole, Perturbation Methods in Applied Mathematics, Springer Verlag, 1985.

MA 5115 - Hopf Algebras
3 0 0 6

Algebras, coalgebras & bialgebras, Convolution algebra, antipode & Hopf algebras, Universal constructions, Tensor algebra, Symmetric & exterior algebras, universal enveloping algebras, Group-likes, primitives & coradical filtration, Structure results. Borel-Hopf and Carter-Milnor-Moore theorems, Hopf algebras for hyperplane arrangements, Connection to affine group schemes & quantum groups.

Texts / References:

M. Sweedler, Hopf algebras, W.A.Benjamin, Inc, NewYork, 1969.

E. Abe, Hopf algebras, CUP, 1980.

C. Kassel, Quantum groups, Springer, 1995.

M.Aguiar, S. Mahajan. Bimonoids for hyperplane arrangements, CUP, 2020

D.Radford. Hopf algebras, World Scientific, 2012.

R. Underwood. Fundamentals of Hopf algebras, Springer, 2015.

M.Aguiar, S. Mahajan. Monoidal functors, species & Hopf algebras, AMS, 2010

MA 5116 - Species and Operads
3 0 0 6

Species. Exponential Species, species of linear orders & other examples. Cauchy, Hadamard & substitution products on species & universal constructions. Power series/Generating function of species Operads. Commutative, associative & Lie operads & other examples, Algebras over operads, Koszul theory of Operads, Species and operads for hyperplane arrangements

Texts / References:

F.Bergeron, G.Labelle, P.Leroux. Combinatorial species & tree-like structures. CUP, 1998.

M.Markl, S.Shnider, J. Stasheff. Operads in Algebra, Topology & Physics, AMS, 2002.

M.Aguiar, S. Mahajan. Bimonoids for hyperplane arrangements, CUP, 2020.

J-L.Loday, B.Vallette. Algebraic operads, Springer, 2012

T.Leinster. Higher operads, Higher categories. CUP, 2004.

M.Aguiar and S.Mahajan. Monoidal Functors, species & Hopf algebras, AMS, 2010.

MA 5113 Category Theory I 3 0 0 6

Categories, functors, natural transformations.

Limits and colimits.

Adjoint functors and universal constructions.

Functor categories, comma categories, quotient categories.

Cauchy completeness, Karoubi envelopes.

Cartesian categories, group objects.

The above concepts can be motivated and discussed by connecting them to other areas of mathematics depending on the interests of the instructor and students.

Texts / References:

Aguiar and Mahajan, Monoidal functors, species and Hopf algebras, American Mathematical Society, 2010.

Awodey, Category theory, Oxford University Press, 2010.

Borceau, Handbook of categorical algebra, Volumes 1, 2, 3, Cambridge University Press, 1994.

Leinster, Higher categories, Higher operads, Cambridge University Press, 2004.

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MA 5118 Category Theory II 3 0 0 6

Monoidal categories, monoids, comonoids.

Symmetric monoidal categories, braidings, Hopf monoids.

Higher monoidal categories.

2-categories, bicategories, higher categories.

Monads, distributive laws, higher monads.

The above concepts can be motivated and discussed by connecting them to other areas of mathematics depending on the interests of the instructor and students.

Texts / References:

Aguiar and Mahajan, Monoidal functors, species and Hopf algebras, American Mathematical Society, 2010.

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