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Abstracts of Talks

Approximate roots

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Certain exponents in Newton's fractional power series expansion lead to characteristic sequences, whose importance was recognized by Smith in 1873 and Halphen in 1884. The theory of approximate roots gives a more direct approach to these sequences. The approximates roots of a poloynomial are defined by generalizing the completing the square method of solving quadratic equations put forth by Shreedharacharya in 700 A.D. and versified by Bhaskaracharya in his 1150 A.D. algebra book Beejaganit. A combination of both these methods provides an effective tool for studying various questions of affine algebraic geometry such as the epimorphism theorem, the automorphism theorem, and the jacobian conjecture.

Associated primes and numerical invariants of graded local cohomology modules

Markus Brodmann, Universität Zürich, Switzerland.

Let $R = R_0 + R_1 + \cdots$ be a Noetherian homogeneous ring and let M he a finitely generated graded R-module. Let i be a non-negative integer. We give a survey on some results on the asymptotic behaviour of the n-the graded component $G(n) := H^i_{R_+}(M)_n$ of the i-th local cohomology module of M supported in the irrelevant ideal R_+ of R for $n << 0$. In particular, we report on the following results, obtained in recent joint work with S. Fumasoli, S. Lim and F. Rohrer:

(1) If the base ring R_0 is a domain, it contains some non-zero element such that the $(R_0)_s$-modules $G(n)_s$ are torsion-free for all integers n.
(2) If the base ring R_0 is semilocal and of dimension at most two and essentially of finite type over a field, the set of associated primes $\mathrm{Ass}_{R_0}(G(n))$ of the above modules is asymptotically stable for $n << 0$.
(3) If the base ring R_0 has dimension one and is essentially of finite type over a field, the same holds true.
(4) If (R_0, m_0) is local and of dimension 1, much can be said on the Hilbert-Samuel polynomials of the above modules $G(n)$: for an arbitrary m_0-primary ideal q_0, the (two) Hilbert-Samuel coefficients of $G(n)$ with respect to q_0 are antipolynomial functions of degree $< i$ in n, whereas the corresponding postulation numbers of the modules $G(n)$ have a common upper bound.

We discuss some relations between these results and examples of M. Katzman, and I. Swanson-A. Singh.

Cohen-Macaulay and Ulrich ideals of determinantal rings

Winfried Bruns, Universität Osnabrück, Germany.

We study initial algebras of determinantal rings, defined by minors of generic matrices, with respect to their classical generic point. This approach leads to very short proofs for the structural properties of determinantal rings. Moreover, it allows us to classify their Cohen-Macaulay and Ulrich ideals. (Joint work with T. Römer and A. Wiebe)

Regularity of ideals and their powers

Marc Chardin, Université de Paris VI, France.

I will present recent result of two types: Bounds on the Castelnuovo-Mumford regularity in terms of degrees of defining equations (notably the cases of generically reduced and generically complete intersection schemes of dimension two), and regularity bounds for the powers of an ideal in terms of the regularity of the ideal itself and the maximal degree of a minimal generator.

Poincaré series of singularities

Steven Dale Cutkosky, University of Missouri-Columbia, USA.

We compute the Poincare Series of a generally non-noetherian graded ring associated to divisors on a singularity or variety, and consider the question of when this series is rational. We give examples showing that in general the Poincare series is not rational, and give some criteria ensuring that it is rational. (Joint work with Ana Reguera and Juergen Herzog)

Some results on subalgebras of polynomial algebras

Amartya Kumar Dutta, Indian Statistical Institute, Kolkata, India.

We shall discuss some results pertaining to an R-subalgebra A of the polynomial ring $R[X]$ over a noetherian integral domain R. When R and A are both UFDs, by a result of Abhyankar-Eakin-Heinzer, A is necessarily a polynomial ring over R. This result does not hold over more general domains. However, some surprisingly mild conditions on merely the generic and codimension one fibres of the R-algebra A ensure that A is a symmetric algebra of an invertible ideal of R (when R is normal) or at least an affine fibration (when R is more general). A crucial technical ingredient in these results — which could be of independent interest — is a patching lemma involving the splitting of certain general linear automorphisms. We shall also discuss some sufficient fibre conditions for finite generation of a noetherian R-subalgebra of $R[X]$ over a locally factorial domain R.

Completions of rank 1 valuation rings

Laura Ghezzi, University of Missouri-Columbia, USA.

This is joint work with S.D. Cutkosky. Let k be a field of characteristic zero, K an algebraic function field over k, and V a k-valuation ring of K. Zariski's theorem of local uniformization shows that there exist algebraic regular local rings R_i with quotient field K which are dominated by V, and such that the direct limit $\cup R_i = V$. We investigate the ring $T = \cup \hat{R}_i$. The ring T is Henselian and thus can be considered to be a "completion" of the valuation ring V. We give an example showing that T is in general not a valuation ring. Making use of a result of Heinzer and Sally, we give necessary and sufficient conditions for T to be a valuation ring. The essential obstruction to T being a valuation ring is the problem of the rank of the valuation increasing upon extending the valuation dominating a particular R to a valuation dominating its completion. In the case of rank 1 valuations, this problem can be handled in a very satisfactory way. Finally, suppose that K^* is a finite algebraic extension of K and V^* is a rank 1 k-valuation ring of K^* such that $V = V^* \cap K$. We obtain a relative local uniformization theorem for the extension K^* of K, that generalizes previous results of Cutkosky and Piltant.

Two remarks on the topology of projective surfaces

Rajendra V. Gurjar, Tata Institute of Fundamental Research, Mumbai, India.

We will discuss proofs of two results.
Theorem 1. If the Shafarevich Conjecture on uniformization of smooth projective varieties is true then the second homotopy group of any smooth projective surface is torsion-free.

Theorem 2. Any 2-dimensional function field over the complex numbers has a normal projective model which is simply-connected.

Distributive lattices, bipartite graphs and Alexander duality

Jürgen Herzog, Universität Essen, Germany.

This is a report on joint work with Takayuki Hibi. A certain squarefree monomial ideal H_P arising from a finite partially ordered set P is studied from viewpoints of both commutative algebra and combinatorics. It is shown that the defining ideal of the Rees algebra of H_P admits a quadratic Gröbner basis. Thus in particular all powers of H_P have linear resolutions. The minimal free graded resolution of H_P will explicitly constructed and a combinatorial formula to compute the Betti numbers of H_P is presented. All the Cohen–Macaulay bipartite graphs are classified, by using the fact that the Alexander dual of the simplicial complex Δ whose Stanley–Reisner ideal coincides with H_P is Cohen-Macaulay.

A Criterion for Regular Sequences

Dilip P. Patil, Indian Institute of Science, Bangalore, India.

(Joint work with U. Storch and J. Stückrad) Let R be a commutative noetherian ring and $f_1, \ldots, f_r \in R$. We give a criterion for $f_1, \ldots, f_r$ to be a (locally) regular sequence for a finitely generated R-module which strengthens and generalises a result in [1]. As an immediate consequence we deduce that if $\mathrm{V}(g_1, \ldots, g_r) \subseteq \mathrm{V}(f_1, \ldots, f_r)$ and if $f_1, \ldots, f_r$ is a (locally) regular sequence in R, then $g_1, \ldots, g_r$ is also a (locally) regular sequence in R. More precisely, we prove the following :

Theorem. Let R be a commutative noetherian ring, $f_1, \ldots, f_r \in R$ and let M be a finitely generated R-module. Then the following statements are equivalent :

(i) $f_1, \ldots, f_r$ is a regular sequence on M.
(ii) $\mathrm{depth}_{R_p}(M_p) \geq r$ for every $p \in \mathrm{Supp}(M/(f_1, \ldots, f_r)M)$.
(iii) $\mathrm{depth}_{R_p}(M_p) \geq r$ for every $p \in \mathrm{Ass}(M/(f_1, \ldots, f_r)M)$.

Corollary 1. (Eisenbud, Herrmann, Vogel [1 ; Corollary 1]) *Let R be a commutative noetherian ring, $f_1, \ldots, f_r \in R$ and let M be a finitely generated R-module. Then $f_1, \ldots, f_r$ is a regular sequence on M if and only if $f_1, \ldots, f_r$ is a regular sequence on M_p for every $p \in \mathrm{Ass}(M/(f_1, \ldots, f_r)M)$.*

Corollary 2. *Let R be a commutative noetherian ring and let $f_1, \ldots, f_r$, $g_1, \ldots, g_r \in R$. Let M be a finitely generated R-module with $\mathrm{Supp}(M/(g_1, \ldots, g_r)M) \subseteq \mathrm{Supp}(M/(f_1, \ldots, f_r)M)$. Suppose that $f_1, \ldots, f_r$ is a regular sequence on M. Then $g_1, \ldots, g_r$ is also a regular sequence on M. In particular, if $\mathrm{V}(g_1, \ldots, g_r) \subseteq \mathrm{V}(f_1, \ldots, f_r)$ and if $f_1, \ldots, f_r$ is a regular sequence in R, then $g_1, \ldots, g_r$ is also a regular sequence in R.*

Corollary 3. *If M is a finitely generated Cohen-Macaulay module over a noetherian local ring R, then every system of parameters of M is a regular sequence on M. In particular, in a Cohen-Macaulay local ring every system of parameters is a regular sequence.*

References

[1] D. Eisenbud, M. Herrmann and W. Vogel, Remarks on regular sequences, *Nagoya Math. J.* **67**, (1977), 177-180.

[2] G. Scheja and U. Storch. "Regular Sequences and Resultants", Research Notes in Mathematics, **8**, A K Peters, Natick, Massachusetts 2001.

Classification of Quadruple canonical covers

Bangere Purnaprajna, University of Kansas, Lawrence, KS, USA.

In this talk I will deal with the recent result (with F. J. Gallego) on classification of quadruple Galois canonical covers of surfaces of minimal degree. The classification shows that they are either non-simple cyclic covers or bi-double covers. If they are bi-double then with one exception, they are all fiber products of double covers. We construct examples to show that all the possibilities in the classification do exist. There are implications of this classification that include the existence of families with unbounded p_g and families with unbounded q, in sharp contrast with the case of double and triple canonical covers (due to Horikawa and Konno). Also, it turns out that if the image of the canonical morphism is singular, then the geometric genus of X is in fact bounded by 4. Together with the results of Horikawa and Konno for double and triple covers, a striking numerology emerges that motivates some general questions on the existence of higher degree canonical covers. In this talk, I will also answer some of these questions.

Degenerations of vector bundles on three space

A. Prabhakar Rao, University of Missouri-St. Louis, MO, USA.

We look at explicit examples of degenerations of rank two bundles on projective three space. The goal is to understand how boundary components appear in the closure of the moduli space of stable bundles, where the boundary components may parametrize stable sheaves which are not bundles.

A Fundamental Property of Suslin Matrices

Ravi A. Rao , Tata Institute of Fundamental Research, Mumbai, India.

Given two rows v, w with dot product 1, A. Suslin has constructed a matrix $S_r(v, w)$ of size 2^r and determinant one. These matrices have several interesting properties and uses in problems of set theoretic complete intersections, and classical K-theory. The subgroup generated by them can be related to elementary orthogonal groups, and spinor groups. We sketch this adventure.

Geometric reductivity - revisited

C. S. Seshadri, Chennai Mathematical Institute.

In a paper published in the Annals (in the early 70's), a partial solution of geometric reductivity of semisimple groups was given (this was proved completely, later by Haboush). This talk will centre around remarks on this earlier work of mine.

The Bockstein homomorphism for local cohomology

Anurag Singh, Georgia Institute of Technology, Atlanta, GA, USA.

We investigate the Bockstein sequence for local cohomology modulo arising from reduction module a prime integer. Our interest in this arises from Lyubeznik's conjecture that local cohomology modules of regular rings have finitely many associated prime ideals. The talk is based on work in progress with Uli Walther.

On a question of Huneke concerning seminormality

Balwant Singh, Indian Institute of Technology Bombay, Mumbai, India.

We answer a question of Huneke in the negative by constructing an example of a ring A containing a nonzero divisor t such that A/tA is a seminormal integral domain but A is not seminormal. We also show that this example is universal among such examples.

A problem in linear algebra

Vasudevan Srinivas, Tata Institute of Fundamental Research, Mumbai, India.

A possible generalization of the Serre problem (Quillen-Suslin theorem) on the freeness of projective modules over polynomial rings, which seems to be of interest in systems theory applications, is shown to follow from the solution to the original Serre problem.

Some computations of Hilbert-Kunz functions and multiplicities

Vijaylaxmi Trivedi, Tata Institute of Fundamental Research, Mumbai, India.

In this talk we give a method to compute Hilbert-Kunz functions and multiplicities for certain projective embeddings of full flag varieties and elliptic curves, over algebraically closed fields of positive characteristics. We also indicate an approach to such computations for certain other classes of projective curves, via stability. As a consequence, we deduce the existence of some new classes of stable vector bundles on curves whose Frobenius pull-back is not semi-stable.

The Jacobian Conjecture: formal inverse and recent reductions

David Wright , Washington University in St. Louis, St. Louis, MO, USA

Keller's Jacobian Conjecture has remained unsolved for over sixty years. Hilbert's Nullstellensatz together with Gröbner basis methods tell us that the conjecture is algorithmic for any fixed degree d and dimension n. Reductions using stability have led to studying polynomial maps having certain special forms, and a very recent result of van den Essen reduces the the case when the Jacobian matrix is a Hessian. We will examine the ramifications of these reductions (especially the latter), explain why they point to a possible combinatorial solution to the conjecture, and report on some calculations made with the symbolic algebra programme *Singular.*

On the Rees powers of a module

Santiago Zarzuela, Universitat de Barcelona, Spain.

Let R be a commutative ring and let $G \simeq R^e$ be a free R-module with finite rank $e > 0$. For any R-submodule $E \subset G$ one may consider the image of the symmetric algebra of E by the natural map to the symmetric algebra of G, and then the graded components E_n, $n \geq 0$ of this image, that we shall call the Rees powers of the module E (with respect to the embedding $E \subset G$). In this talk I shall discuss some assymptotic properties of the R-modules E_n, $n \geq 0$. (Joint work with A. L. Branco Correia.)