Assignment 1 MA 401
Autumn 2018, IIT Bombay

(1) Find a system of real linear equations whose solution set is the following if it exists.
 (a) $\{(1,1,1)\}$
 (b) $\{(s,-s)\}$
 (c) $\{(1,0,0),(0,1,0),(0,0,1)\}$.

(2) Let F be a field. Show that the following properties are true.
 (a) The additive identity is unique.
 (b) The multiplicative identity is unique.
 (c) Cancellation law holds, that is, $a+b=a+c \implies b=c$ for all $a,b,c \in F$ and $a.b=a.c \implies b=c$ for all $b,c \in F$ and $a \in F-\{0\}$.
 (d) $0.x=0$ for all $x \in F$.

(3) Show that there exists a unique field of two elements.

(4) Show that $\mathbb{Z}_p$ which is the set of integers modulo p is a field when p is prime.

(5) Show that a homogeneous system of m real linear equations in n-real variables has infinitely many solutions if $n > m$.

(6) Find the set of all solutions to system of equations

$$y - x^2 = 0;\ y^2 - x^2 = 0.$$

(7) Show that a consistent system of linear equations always has a either a unique solution or infinitely many solutions. What happens if the set of equations were not linear?

(8) Show that $\mathbb{R}^2$ is a vector space over the field $\mathbb{Q}$.

(9) Show that every field F is a vector space over itself.

(10) State true or false with explanation. A homogeneous system of m linear equations over a field F in n variables taking values in F has infinitely many solutions if $n > m$.