February 5, 2013

Tutorial 4
MA 406 General Topology, IIT Bombay

(1) State true or false. The set $\mathbb{Q}$ as a subspace of $\mathbb{R}$ is a discrete space.

(2) If X is metrizable and $Y \subset X$ then the subspace Y is a metric subspace of X.

(3) Let X be a topological space. Let f and g are continuous functions from $X \to \mathbb{R}$. Show that $\{x \in X \mid f(x) = g(x)\}$ is closed. Let D be a dense subset of X. Show that if $f(x) = g(x)$ for all $x \in D$ then $f = g$.

(4) Show that the following properties are preserved by homeomorphisms $f : X \to Y$.
   (a) X is metrizable.
   (b) X has countably many open sets.
   (c) X is second countable.

(5) Show that $(A \times B)^\circ = A^\circ \times B^\circ$ for subsets A and B of topological spaces X and Y respectively.

(6) Show that $\overline{A \times B} = \overline{A} \times \overline{B}$ for subsets A and B of topological spaces X and Y respectively.