PARTIAL OLUTIONS TO UTORIALS.

(1) Was discussed in class.

(2) Already emailed a sketch of proof

(3). Loser. Look for a graph of a function f. Note the graph is closed in $\mathbb{R}^2$ if f is cts both (show this)

Choose a f such that image of the graph in $\mathbb{R}$ after taking proj onto the first factor is not closed.

(4). Let X_α be homeomorphic to X $\forall \alpha \in \Lambda$.

T.ST $\bigsqcup_{\alpha \in \Lambda} X_\alpha \cong X \times \Lambda$.

Defn Let. f_α denote the homeomorphism

$X_\alpha \to X_\alpha$ $\forall \alpha \in \Lambda$.

Define. $X \times \Lambda \xrightarrow{F} \bigsqcup_{\in \Lambda} X_\alpha$

$(x, \alpha) \xrightarrow{f} f_\alpha(x)$.

verify this is a well defined bijection

T.S.T. this map is an homeomorphism

We need to show that.

$\mathcal{U}$ is open in $X \times \Lambda$ iff $F(\mathcal{U})$ is open X_α.

The topology on $X \times \Lambda$ is the product topology which has base $V \times \{\alpha\}$ since Λ has discrete top.

The topology on $\coprod_{\alpha \in \Lambda} X_\alpha$ is given by as.

A is open in $\coprod_{\alpha \in \Lambda} X_\alpha$ if $\forall$ $A \cap X_\alpha$ is open.

Note now. $F(V \times \{\alpha\}) = f_\alpha(V) \subseteq X_\alpha$.

$\Rightarrow$ $f_\alpha(V) \cap \beta = \phi$ if $\beta \neq \alpha$

$= f_\alpha(V)$ if $- = \alpha$.

But f_α is a homeomorphism) $f_\alpha(V)$ is open.

Claw. nversely if $F(u)$ is open in $\coprod X_\alpha$.

u is open in $X \times \Lambda$.