# Assignment 9 MA 5102
# Autumn 2017, IIT Bombay

(1) Find the CW structure for the following spaces
  (a) $\mathrm{S}^n$.
  (b) $\mathrm{T}^2$.
  (c) $\mathbb{R}\mathrm{P}^3$.
  (d) $\mathbb{C}\mathrm{P}^2$, The space of lines in $\mathbb{C}^3$.

(2) Show that the following spaces are CW complexes.
  (a) The quotient of $\mathrm{S}^2$ obtained by identifying the north and south pole.
  (b) $\mathrm{S}^1 \times (\mathrm{S}^1 \vee \mathrm{S}^1)$.
  (c) The space obtained from $\mathrm{D}^2$ by first deleting the interiors of two disjoint subdisks in the interior of $\mathrm{D}^2$ and then identifying all three resulting boundary circles together via homeomorphisms preserving clockwise orientations of these circles.

(3) Show that $\mathrm{T}^2$ is a simplicial complex, that is, write down its triangulation.

(4) Let $X$ be a space and consider its singular chain complex $(S_n(X), d_n)$. Show that for any abelian group $G$, $(S_n(X) \otimes G, d_n \otimes \mathrm{id}_G)$ is again a chain complex.

(5) Consider the following. Let $C_n = \mathbb{Z}/8$ for $n \geq 0$ and $C_n = 0$ for $n < 0$. Let $d_0 = 0$ and let $d_n : C_n \to C_{n-1}$ map $x \pmod 8$ to $4x \pmod 8$.
  Show that this is a chain complex and compute its homology.

(6) A morphism of chain complexes of groups $\phi : (C_\bullet, d_C) \to (D_\bullet, d_D)$ is a sequence of group homomorphisms $\phi_n : C_n \to D_n$ such that $\phi_n d_C = d_D \phi_n$.
  Show that such map will induce a map on the homology, that is, $\phi_* : H_n(C) \to H_n(D)$.

(7) Let $K$ be a geometric $n$-dimensional simplicial complex and let $K_k$ denote the set of $k$-dimensional simplices of $K$. Let the maps $\delta_i : K_k \to K_{k-1}$ denote the map mapping the $k$-simplex to the its $k-1$-face opposite to the $i^{th}$ vertex.
  Define $C_k$ to be free abelian group on $K_k$ and $C_k = 0$ for $k < 0$ and $k > n$. Define the linear map on $d_k : C_k \to C_{k-1}$ as $d_k = \sum_{i=0}^{k} (-1)^i \delta_i$.
  Prove that $(C_k, d_k)$ defines a chain complex.