MA 2017, Tutorial Sheet-3
Frobenius method and regular singular points

1. Attempt a power series solution around $x = 0$ for $\quad x^2y'' - (1 + x)y = 0.$

   Explain why the procedure does not give any nontrivial solutions.

2. Attempt a Frobenius series solution for the differential equation

   $x^2y'' + (3x - 1)y' + y = 0. \qquad$ Why does the method fail?

3. Locate and classify the singular points for the following differential equations.

   (All letters other than x and y such as p, λ, etc are constants.)

   (a) Bessel equation: $\quad x^2y'' + xy' + (x^2 - p^2)y = 0.$

   (b) Laguerre equation: $\quad xy'' + (1 - x)y' + \lambda y = 0.$

   (c) Jacobi equation: $\quad x(1 - x)y'' + (\gamma - (\alpha + 1)x)y' + n(n + \alpha)y = 0.$

   (d) Hypergeometric equation: $\quad x(1 - x)y'' + [c - (a + b + 1)x)]y' - aby = 0.$

   (e) Associated Legendre equation: $\quad (1 - x^2)y'' - 2xy' + \left[n(n + 1) - \dfrac{m^2}{1 - x^2}\right] y = 0$

   (f) $xy'' + (\cot x)y' + xy = 0.$

4. In (3), find the indicial equations corresponding to all the regular singular points.

5. Find two linearly independent solutions around $x = 0$ of the following differential equations.

   (a) $x^2y'' + x\frac{2x-1}{2}y' + \frac{1}{2}y = 0.$

   (b) $x^2y'' + x(x^2 - 3)y' + (4 + x^2)y = 0.$

   (c) $x^2y'' + x\frac{2x-1}{2(1+x)}y' + \frac{1}{2(1+x)}y = 0.$

   (d) $x^2y'' - x(2 - x^2)y' + (2 + x^2)y = 0.$

   (e) $x^2(2 - x^2)y'' - 2x(1 + 2x^2)y' + (2 - 2x^2)y = 0.$

   (f) $x^2(1 + x^2)y'' + x(3 + 10x^2)y' - (15 - 14x^2)y = 0.$