MA 2017, Tutorial Sheet-7
Wave equation and Laplace equation

1. Solve the following wave equations.

(a) $u_{tt} = 9u_{xx}, \quad 0 < x < 1,\ t > 0,$
$u(0,t) = 0 = u(1,t), \quad t \geq 0$
$u(x,0) = x(1-x), \quad u_t(x,0) = 0,\ 0 \leq x \leq 1.$

(b) $u_{tt} = 9u_{xx}, \quad 0 < x < 1,\ t > 0,$
$u(0,t) = 0 = u(1,t), \quad t \geq 0$
$u(x,0) = 0, \quad u_t(x,0) = x(1-x),\ 0 \leq x \leq 1.$

(c) $u_{tt} = 4u_{xx}, \quad 0 < x < 1,\ t > 0,$
$u(0,t) = 0 = u(1,t), \quad t \geq 0$
$u(x,0) = 0, \quad u_t(x,0) = x(x^3 - 2x^2 + 1),\ 0 \leq x \leq 1.$

(d) $u_{tt} = 5u_{xx}, \quad 0 < x < \pi,\ t > 0,$
$u(0,t) = 0 = u(\pi,t), \quad t \geq 0$
$u(x,0) = x\sin x, \quad u_t(x,0) = 0,\ 0 \leq x \leq \pi.$

(e) $u_{tt} = 5u_{xx}, \quad 0 < x < 2,\ t > 0,$
$u_x(0,t) = 0 = u_x(2,t), \quad t \geq 0$
$u(x,0) = 2x^2(3-x), \quad u_t(x,0) = 0,\ 0 \leq x \leq 2.$

(f) $u_{tt} = 5u_{xx}, \quad 0 < x < 2,\ t > 0,$
$u_x(0,t) = 0 = u_x(2,t), \quad t \geq 0$
$u(x,0) = 0, \quad u_t(x,0) = 2x^2(3-x),\ 0 \leq x \leq 2.$

(g) $u_{tt} = 16u_{xx}, \quad 0 < x < \pi,\ t > 0,$
$u_x(0,t) = 0 = u_x(\pi,t), \quad t \geq 0$
$u(x,0) = x^2(x-\pi)^2, \quad u_t(x,0) = 0,\ 0 \leq x \leq \pi.$

(h) $u_{tt} = 16u_{xx}, \quad 0 < x < \pi,\ t > 0,$
$u_x(0,t) = 0 = u_x(\pi,t), \quad t \geq 0$
$u(x,0) = 0, \quad u_t(x,0) = x^2(x-\pi)^2, \quad 0 \leq x \leq \pi.$

2. Solve the following Laplace equations.

(a) $u_{xx} + u_{yy} = 0, \quad 0 < x < 1,\ 0 < y < 1,$
$u(x,0) = x(1-x), \quad u(x,1) = 0, \quad 0 \leq x \leq 1,$
$u(0,y) = 0, \quad u(1,y) = 0, \quad 0 \leq y \leq 1.$

(b) $u_{xx} + u_{yy} = 0, \qquad 0 < x < 2,\ 0 < y < 3,$

$u(x,0) = x^2(2-x),\ \ u(x,3) = 0, \qquad 0 \le x \le 2$

$u(0,y) = 0,\ \ u(2,y) = 0 \qquad 0 \le y \le 3.$

(c) $u_{xx} + u_{yy} = 0, \qquad 0 < x < \pi,\ 0 < y < \pi,$

$u(x,0) = x\sin x,\ \ u(x,\pi) = 0, \qquad 0 \le x \le \pi,$

$u(0,y) = 0,\ \ u(\pi,y) = 0 \qquad 0 \le y \le \pi.$

(d) $u_{xx} + u_{yy} = 0, \qquad 0 < x < 2,\ 0 < y < 2,$

$u(x,0) = 0,\ \ u(x,2) = x^2 - 4, \qquad 0 \le x \le 2$

$u_x(0,y) = 0,\ \ u_x(2,y) = 0, \qquad 0 \le y \le 2$

(e) $u_{xx} + u_{yy} = 0, \qquad 0 < x < 2,\ 0 < y < 1,$

$u_y(x,0) = 0,\ \ u_y(x,1) = 0, \qquad 0 \le x \le 2$

$u(0,y) = y^2(3-2y),\ \ u(2,y) = 0, \qquad 0 \le y \le 2.$

(f) $u_{xx} + u_{yy} = 0, \qquad 0 < x < 2,\ 0 < y < 3,$

$u(x,0) = 0,\ \ u(x,3) = 0, \qquad 0 \le x \le 2$

$u_x(0,y) = 0,\ \ u_x(2,y) = y(3-y),\ 0 \le y \le 3.$