

MA207 - Tutorial Sheet 3

August 24, 2021

1. Attempt a power series solution around $x = 0$ for $x^2y'' - (1 + x)y = 0$.

Explain why the procedure does not give any nontrivial solutions.

2. Attempt a Frobenius series solution for the differential equation

$$x^2y'' + (3x - 1)y' + y = 0. \quad \text{Why does the method fail?}$$

3. Locate and classify the singular points for the following differential equations.

(All letters other than x and y such as p , λ , etc are constants.)

(a) Bessel equation: $x^2y'' + xy' + (x^2 - p^2)y = 0$.

(b) Laguerre equation: $xy'' + (1 - x)y' + \lambda y = 0$.

(c) Jacobi equation: $x(1 - x)y'' + (\gamma - (\alpha + 1)x)y' + n(n + \alpha)y = 0$.

(d) Hypergeometric equation: $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$.

(e) Associated Legendre equation: $(1 - x^2)y'' - 2xy' + \left[n(n + 1) - \frac{m^2}{1 - x^2} \right] y = 0$

(f) $xy'' + (\sin x)y' + xy = 0$.

4. In (3), find the indicial equations corresponding to all the regular singular points.

5. Find two linearly independent solutions around $x = 0$ of the following differential equations.

(a) $x^2y'' + \frac{x(2x - 1)}{2}y' + \frac{1}{2}y = 0$.

(b) $x^2y'' + x(x^2 - 3)y' + (4 + x^2)y = 0$.

(c) $x^2y'' + \frac{x(2x - 1)}{2(1 + x)}y' + \frac{1}{2(1 + x)}y = 0$.

(d) $x^2y'' - x(2 - x^2)y' + (2 + x^2)y = 0$.

(e) $x^2(2 - x^2)y'' - 2x(1 + 2x^2)y' + (2 - 2x^2)y = 0$.

(f) $x^2(1 + x^2)y'' + x(3 + 10x^2)y' - (15 - 14x^2)y = 0$.