

MA207 - Tutorial Sheet 5

August 24, 2021

1. Show that
$$\sum_{n=1}^{\infty} \frac{(\sin n\alpha)^2}{n} \sin nx = \begin{cases} \pi/4 & 0 < x < 2\alpha \\ 0 & 2\alpha < x < \pi \end{cases}$$
2. Show that
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \cos nx = \frac{\pi^2}{12} - \frac{x^2}{4}, \quad -\pi \leq x \leq \pi.$$
3. Show that
$$\sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{(2n+1)^3} = \frac{1}{8}\pi x(\pi-x), \quad 0 \leq x \leq \pi.$$
4. Use the Fourier expansions given in problems (1), (2) and (3) along with Fourier's Theorem to deduce the following results. (Plug in specific values of α and x .)
 - (a) $1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{5} + \frac{1}{7} + \frac{1}{8} - \frac{1}{10} - \frac{1}{11} + \dots = \frac{2\pi}{3\sqrt{3}}$
 - (b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{1}{7} - \frac{1}{8} + \frac{1}{10} - \frac{1}{11} + \dots = \frac{\pi}{3\sqrt{3}}$
 - (c) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \dots = \frac{\pi^2}{12}$
 - (d) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots = \frac{\pi^2}{6}$ (Euler's formula)
 - (e) $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} - \dots = \frac{\pi^3}{32}$
 - (f) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \dots = \frac{\pi^2}{8}$
 - (g) $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{\pi}{4} - \frac{1}{2}$
5. Find the Fourier series of $f(x)$ on $[-L, L]$ and determine the value that the series takes for $-L \leq x \leq L$.
 - (a) $L = \pi, f(x) = 2x - 3x^2$
 - (b) $L = 1, f(x) = 1 - 3x^2$
 - (c) $L = \pi, f(x) = |\sin x|$
6. Expand each of the following functions in a Fourier cosine series on $[0, L]$.

- (a) $L = 1, f(x) = e^{-x}$
- (b) $L = 2, f(x) = \begin{cases} 0, & 0 \leq x \leq 1 \\ 1, & 1 \leq x \leq 2 \end{cases}$
- (c) $L = \pi, f(x) = 2 \sin x \cos x$
- (d) $f(x) = x^2 - L^2$
- (e) $f(x) = 3x^2(x^2 - 2L^2)$
- (f) $f(x) = x^3(3x - 4L)$
- (g) $x^2(3x^2 - 8Lx + 6L^2)$

7. Expand each of the following functions in a Fourier sine series on $[0, L]$.

- (a) $L = 1, f(x) = e^{-x}$
- (b) $L = 2a, f(x) = \begin{cases} x, & 0 < x < a \\ a, & a \leq x \leq 2a \end{cases}$
- (c) $L = \pi, f(x) = 2 \sin x \cos x$
- (d) $L = \pi, f(x) = \cos x$
- (e) $f(x) = x(L^2 - x^2)$
- (f) $f(x) = x(x^3 - 2Lx^2 + L^3)$
- (g) $f(x) = x(3x^4 - 5Lx^3 + 2L^4)$