

MA207 - Tutorial Sheet 7

August 24, 2021

1. Solve the following wave equations.

- (a) $u_{tt} = 9u_{xx}, \quad 0 < x < 1, t > 0,$
 $u(0, t) = 0 = u(1, t), \quad t \geq 0$
 $u(x, 0) = x(1 - x), \quad u_t(x, 0) = 0, 0 \leq x \leq 1.$
- (b) $u_{tt} = 9u_{xx}, \quad 0 < x < 1, t > 0,$
 $u(0, t) = 0 = u(1, t), \quad t \geq 0$
 $u(x, 0) = 0, \quad u_t(x, 0) = x(1 - x), 0 \leq x \leq 1.$
- (c) $u_{tt} = 4u_{xx}, \quad 0 < x < 1, t > 0,$
 $u(0, t) = 0 = u(1, t), \quad t \geq 0$
 $u(x, 0) = 0, \quad u_t(x, 0) = x(x^3 - 2x^2 + 1), 0 \leq x \leq 1.$
- (d) $u_{tt} = 5u_{xx}, \quad 0 < x < \pi, t > 0,$
 $u(0, t) = 0 = u(\pi, t), \quad t \geq 0$
 $u(x, 0) = x \sin x, \quad u_t(x, 0) = 0, 0 \leq x \leq \pi.$
- (e) $u_{tt} = 5u_{xx}, \quad 0 < x < 2, t > 0,$
 $u_x(0, t) = 0 = u_x(2, t), \quad t \geq 0$
 $u(x, 0) = 2x^2(3 - x), \quad u_t(x, 0) = 0, 0 \leq x \leq 2.$
- (f) $u_{tt} = 5u_{xx}, \quad 0 < x < 2, t > 0,$
 $u_x(0, t) = 0 = u_x(2, t), \quad t \geq 0$
 $u(x, 0) = 0, \quad u_t(x, 0) = 2x^2(3 - x), 0 \leq x \leq 2.$
- (g) $u_{tt} = 16u_{xx}, \quad 0 < x < \pi, t > 0,$
 $u_x(0, t) = 0 = u_x(\pi, t), \quad t \geq 0$
 $u(x, 0) = x^2(x - \pi)^2, \quad u_t(x, 0) = 0, 0 \leq x \leq \pi.$
- (h) $u_{tt} = 16u_{xx}, \quad 0 < x < \pi, t > 0,$
 $u_x(0, t) = 0 = u_x(\pi, t), \quad t \geq 0$
 $u(x, 0) = 0, \quad u_t(x, 0) = x^2(x - \pi)^2, 0 \leq x \leq \pi.$

2. Solve the following Laplace equations.

$$\begin{aligned} \text{(a)} \quad & u_{xx} + u_{yy} = 0, & 0 < x < 1, 0 < y < 1, \\ & u(x, 0) = x(1 - x), \quad u(x, 1) = 0, & 0 \leq x \leq 1, \\ & u(0, y) = 0, \quad u(1, y) = 0, & 0 \leq y \leq 1. \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & u_{xx} + u_{yy} = 0, & 0 < x < 2, 0 < y < 3, \\ & u(x, 0) = x^2(2 - x), \quad u(x, 3) = 0, & 0 \leq x \leq 2 \\ & u(0, y) = 0, \quad u(2, y) = 0 & 0 \leq y \leq 3. \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & u_{xx} + u_{yy} = 0, & 0 < x < \pi, 0 < y < \pi, \\ & u(x, 0) = x \sin x, \quad u(x, \pi) = 0, & 0 \leq x \leq \pi, \\ & u(0, y) = 0, \quad u(\pi, y) = 0 & 0 \leq y \leq \pi. \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & u_{xx} + u_{yy} = 0, & 0 < x < 2, 0 < y < 2, \\ & u(x, 0) = 0, \quad u(x, 2) = x^2 - 4, & 0 \leq x \leq 2 \\ & u_x(0, y) = 0, \quad u_x(2, y) = 0, & 0 \leq y \leq 2 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & u_{xx} + u_{yy} = 0, & 0 < x < 2, 0 < y < 1, \\ & u_y(x, 0) = 0, \quad u_y(x, 1) = 0, & 0 \leq x \leq 2 \\ & u(0, y) = y^2(3 - 2y), \quad u(2, y) = 0, & 0 \leq y \leq 1. \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad & u_{xx} + u_{yy} = 0, & 0 < x < 2, 0 < y < 3, \\ & u(x, 0) = 0, \quad u(x, 3) = 0, & 0 \leq x \leq 2 \\ & u_x(0, y) = 0, \quad u_x(2, y) = y(3 - y), & 0 \leq y \leq 3. \end{aligned}$$