

Ordinary Differential Equations**Homework 3****Important**

- Write your solutions neatly and submit it on **25 October**(tutorials). Late submission will not be allowed.
  - Simplify all your answers as much as possible and express answers in terms of fractions or constants such as  $\sqrt{e}$  or  $\ln(4)$  rather than decimals.
  - Show all your work and explain your reasonings clearly! Copying will not be tolerated.
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1. Find the general solution of the following linear systems.

(a)  $X'(t) = \begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix} X(t),$

(b)  $X'(t) = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} X(t),$

(c)  $X'(t) = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix} X(t).$

2. Solve the initial value problems

(a)  $X'(t) = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} X(t),$  with  $X(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$

(b)  $X'(t) = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} X(t),$  with  $X(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

3. Find the general solution of the ODE:  $x'' + 2x' + x = t^2.$

4. Find the general solution of the ODE:  $X'(t) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} X(t),$  where  $a + d \neq 0$  and  $ad = bc.$

5. Find the limit  $\lim_{t \rightarrow +\infty} X(t)$  where  $X(t)$  satisfies:  $X'(t) = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix} X(t).$

6. Determine the values of  $a$  and  $b$  such that the linear system:  $X'(t) = \begin{pmatrix} a & -b \\ b & 2 \end{pmatrix} X(t)$  has a sink at the origin.

7. Let  $A$  be a  $2 \times 2$  matrix with real entries. Show that:  $\text{Det}(\exp(tA)) = e^{t(\text{Trace}A)}$  for all  $t \in \mathbb{R}.$

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