Ordinary Differential Equations

Problem Set 2

1. Find the general solution of the ODEs:
   (a) $\dfrac{dx}{dt} = x \log\left(\dfrac{1}{x}\right)$.
   (b) $\dfrac{dx}{dt} = \left(\dfrac{t+x+1}{t+2}\right) - \exp\left(\dfrac{t+x+1}{t+2}\right)$.
   (c) $\dfrac{dx}{dt} = \dfrac{t+2x+1}{2t+x+2}$.
   (d) $\dfrac{dx}{dt} = 3|x|^{2/3}$.
   (e) $\dfrac{dx}{dt} = (t-x+3)^2$.
2. Solve the following initial value problems:
   (a) $\dfrac{dx}{dt} = \dfrac{t}{1+t}x + 1$, with the initial condition: $x(0) = 0$.
   (b) $\dfrac{dx}{dt} = \dfrac{e^{-x^2}}{x(2t+t^2)}$, with the initial condition: $x(2) = 0$.
   (c) $\dfrac{dx}{dt} = \dfrac{\cos t}{\cos^2 x}$, with the initial condition: $x(\pi) = \dfrac{\pi}{4}$.
3. Give a first order ODE for the curve $y = cx^2$, where $c \in \mathbb{R}$ is a parameter.
4. Transform the ODE
$$t^2\frac{d^2x}{dt^2} + 3t\frac{dx}{dt} + x = \frac{2}{t},$$
to the new coordinates $y = x$, $s = \log t$. You need not solve it.
5. Show that the ODE
$$\frac{dx}{dt} = t^{n-1} f\left(\frac{x}{t^n}\right)$$
can be solved using the new variable $y = \dfrac{x}{t^n}$.
6. Solve the following ODE
$$x\frac{dy}{dx} + 3x = 2y$$
using an integrating factor $\mu(x)$.