Ordinary Differential Equations

Problem Set 3

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1. Write out the first few terms of the **Picard iterations** for each of the initial value problems. Where possible, find explicit solutions.

   (a) $\dfrac{dx}{dt} = x + 2$, with the initial condition: $x(0) = 2$.

   (b) $\dfrac{dx}{dt} = \cos x$, with the initial condition: $x(0) = 0$.

   (c) $\dfrac{dx}{dt} = x^{4/3}$, with the initial condition: $x(0) = 0$.

   (d) $\dfrac{dx}{dt} = x^{4/3}$, with the initial condition: $x(0) = 1$.

2. Are the following functions **Lipschitz** continuous near $x = 0$.

   (a) $f(x) = \sin\left(\dfrac{2}{x}\right)$.

   (b) $f(x) = \dfrac{1}{2 - x^2}$.

   (c) $f(x) = \sqrt{x} + 2x$.

   (d) $f(x) = x^2 \cos\left(\dfrac{1}{x}\right)$.