Ordinary Differential Equations

Problem Set 4

---

1. Solve the following equations:
   (a) $\dfrac{dx}{dt} = x + \sin t$,
   (b) $\begin{cases} \dot{x} = -y - t \\ \dot{y} = x + t \end{cases}$ with the initial condition: $x(0) = 1, y(0) = 0$,
   (c) $X'(t) = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} X(t)$,
   (d) $X'(t) = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 5t \end{pmatrix}$,
   (e) $X'(t) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} X(t)$,
   (f) $\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} + x = 0$,
   (g) $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 2x = e^t$.
2. Determine if the linear system
$$X'(t) = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} X(t)$$
   has a sink, source, saddle or center at the origin. Also determine the stability of the system.
3. Consider the second order equation $x'' + bx' + cx = 0$, where $a, b \in \mathbb{R}$ are constants. Transform the given equation into a first order system and then find the eigenvalues of the coefficient matrix A. Show that the eigenvalues of A satisfies the equation $z^2 + bz + c = 0$.
