# Ordinary Differential Equations

## Problem Set 6

1. Find the eigenvalues and eigenfunctions of following the boundary value problems:

   (a) $u'' + \lambda u = 0$ on $(0, 1)$, with $u(0) = 0$, $u'(1) + u(1) = 0$.

   (b) $u'' + \lambda u = 0$ on $(0, 2\pi)$, with $u(0) = u(2\pi)$, $u'(0) = u'(2\pi)$.

2. Let
$$\mathcal{L}u := (p(x)u')' + q(x)u.$$

   Here $p(x)$ and $q(x)$ are continuously differentiable functions in $[a, b]$.

   Show that for any $\phi, \psi \in C^2[a, b]$ with $\phi'(a) = \phi(b) = 0$, $\psi(a) = \psi'(b) = 0$ we have:

   $$\int_a^b \phi \mathcal{L}\psi \ dx = \int_a^b \psi \mathcal{L}\phi \ dx.$$

   We say that $\mathcal{L}$ is self-adjoint.