On a question of Suslin about completion of unimodular rows

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Abstract

R.G. Swan and J. Towber showed that if (a^2, b, c) is a unimodular row over any commutative ring R then it can be completed to an invertible matrix over R. This was strikingly generalised by A.A. Suslin who showed that if $(a_0^{r!}, a_1, \ldots, a_r)$ is a unimodular row over R then it can be completed to an invertible matrix. As a consequence A.A. Suslin proceeds to conclude that if $\frac{1}{r!} \in R$, then a unimodular row $v(X) \in Um_{r+1}(R[X])$ of degree one, with $v(0) = (1, 0, \ldots, 0)$, is completable to an invertible matrix. Then he asked
$(\mathbf{S_r(R)})$: Let R be a local ring such that $r! \in GL_1(R)$, and let $p = (f_0(X), \ldots, f_r(X)) \in Um_{r+1}(R[X])$ with $p(0) = e_1(= (1, 0, \ldots, 0))$. Is it possible to embed the row p in an invertible matrix?

Due to Suslin, one knows answer to this question when $r = d + 1$, without the assumption $r! \in GL_1(R)$. In 1988, Ravi Rao answered this question in the case when $r = d$.

In this talk we will discuss about the Suslin's question $S_r(R)$ when $r = d - 1$. We will also discuss about two important ingredients; "homotopy and commutativity principle" and "absence of torsion in $\frac{Um_{d+1}(R[X])}{E_{d+1}(R[X])}$", to answer Suslin's question in the case when $r = d - 1$, where d is the dimension of the ring.