NUMERICAL ASPECTS OF EPSILON MULTIPLICITY

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Suppose that I is an ideal in a Noetherian local ring R with maximal ideal $\mathfrak{m}$. Then the *epsilon multiplicity* of I is defined to be

$$\varepsilon(I) := \limsup_{n\to\infty} \frac{l_R\left(H^0_{\mathfrak{m}}\left(R/I^n\right)\right)}{n^d/d!}.$$

The ε-multiplicity can be seen as a generalization of the classical Hilbert-Samuel multiplicity. In the first talk, we will try to sketch Cutkosky's proof, based on a principle of Okounkov, that $\varepsilon(I)$ exists as a limit in an analytically unramified local ring R. In the second talk we will describe some results of Herzog-Trung and Herzog-Puthenpurakal-Verma which together show that the ε-multiplicity of a monomial ideal in a polynomial ring is a rational number. If time permits we will also discuss some extensions of these results. In the third talk we will discuss some examples by Cutkosky and others where it is shown that ε-multiplicity can be an irrational number. Such examples are motivated by certain constructions in algebraic geometry.