

## ON COEFFICIENT IDEALS

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### ABSTRACT

Let  $(A, \mathfrak{m})$  be a Cohen-Macaulay local ring of dimension  $d \geq 2$  with infinite residue field and let  $I$  be an  $\mathfrak{m}$ -primary ideal. Let For  $0 \leq i \leq d$  let  $I_i$  be the  $i^{\text{th}}$ -coefficient ideal of  $I$ . Also let  $\tilde{I} = I_d$  denote the Ratliff-Rush closure of  $I$ . Let  $G = G_I(A)$  be the associated graded ring of  $I$ . We show that if  $\dim H_{G_+}^j(G)^\vee \leq j-1$  for  $1 \leq j \leq i \leq d-1$  then  $(I^n)_{d-i} = \tilde{I}^n$  for all  $n \geq 1$ . In particular if  $G$  is generalized Cohen-Macaulay then  $(I^n)_1 = \tilde{I}^n$  for all  $n \geq 1$ . As a consequence we get that if  $A$  is an analytically unramified domain with  $G$  generalized Cohen-Macaulay, then the  $S_2$ -ification of the Rees algebra  $A[It]$  is  $\bigoplus_{n \geq 0} \tilde{I}^n$ .

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*Date:* September 18, 2022.

*2020 Mathematics Subject Classification.* Primary 13A30, 13D45 ; Secondary 13H10, 13H15.

*Key words and phrases.* multiplicity, reduction, Hilbert polynomial, associated graded rings, coefficient ideals.