Abstract

In the two lectures in the week March 25-29, 2019, I shall present the details of the construction from the research paper : **Larfeldt and Lech :** Analytic ramifications and at couples of local rings, *Acta Math.* **146** (1981), no. 3-4, 201-208.

Theorem *Let* $\varphi : (A, \mathfrak{m}_A) \longrightarrow (B, \mathfrak{m}_B)$ *be a flat homomorphism of local rings with* $\dim A = \dim B$. *Then there exists a local ring* $(C, \mathfrak{m}_C)$ *and flat local homomorphisms*

$f : (A, \mathfrak{m}_A) \longrightarrow (C, \mathfrak{m}_C)$ *and* $g : (B, \mathfrak{m}_B) \longrightarrow (C, \mathfrak{m}_C)$ *such that :*

(i) *the diagram*

$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[d, "f"'] & B \arrow[d, "g"] \\ C \arrow[r, "\text{nat}"] & \widehat{C} \end{tikzcd}$$

is commutative.

(ii) $\mathfrak{p} := \mathfrak{m}_A C$ *is a prime ideal of* C *with* $\dim C/\mathfrak{p} = 1$.

(iii) $\mathfrak{p}^* := \mathfrak{m}_B \widehat{C}$ *is a prime ideal of* $\widehat{C}$ *with* $\dim \widehat{C}/\mathfrak{p}^* = 1$.

(iv) $\mathfrak{p}^* \cap C = \mathfrak{p}$.

(v) *the map* $g : B \longrightarrow \widehat{C}$ *has the form* $B \xrightarrow{\text{nat. inclusion}} \widehat{B}[\![X]\!]$, *where* X *is an indeterminate.*

It induces a transformation $(A\,, B) \longmapsto \{C\,, \mathfrak{p}\}$ such that the analytic ramification of $\mathfrak{p}$ reflects the structure of $(A\,, B)$ in as much as there exists a commutative diagram :

$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[d, "f"'] & B \arrow[d, "g"] \\ C_{\mathfrak{p}} \arrow[r, "\text{nat}"] & \widehat{C}_{\mathfrak{p}} \end{tikzcd}$$

with unramified flat homomorphisms as vertical maps. This ingenious construction has an interesting applications.