

Seminar on Algebraic Stacks and Moduli Theory

Algebraic stacks are new kinds of ‘spaces’ in algebraic geometry, which are successors to the schemes of Grothendieck. They were invented in 1974 by Michael Artin, and today they have become a standard piece of mathematical technology that young algebraic geometers, number theorists, and even mathematical physicists need to learn and master.

Moduli spaces (schemes) in algebraic geometry were constructed from 1960s onwards principally by using scheme theory and the Geometric Invariant Theory of Mumford (though analytic approaches are also possible over complex numbers). The moduli schemes do a ‘continuous classification’ of mathematical objects of a given kind, which can vary ‘continuously’. Moduli spaces have been tremendously useful in algebraic geometry, number theory and mathematical physics.

What algebraic stacks have to do with moduli. The points of a moduli scheme represent isomorphism classes (or often somewhat larger equivalence classes) of the desired kind of objects. But often these objects have non-trivial automorphism groups, which moreover can vary from object to object. Moduli spaces as schemes lose all information of the automorphism groups, which is vital information. If moduli are constructed as algebraic stacks, then we retain this information in a usable way. The moduli stacks have better functorial properties than moduli schemes, making it easier to do geometry on them. In fact, this is what motivated the development of algebraic stacks, and it is their main use. Hence it is a good idea to study together algebraic stacks and moduli theory.

This seminar aims to introduce young algebraic geometers to the theory of algebraic stacks and moduli spaces as stacks. The only prerequisite is some familiarity with the basic definitions about schemes (e.g. as in Chapter 2 sections 1 to 5 of Hartshorne’s ‘Algebraic Geometry’). If there is an active participation of the audience, then lectures on some of the topics can be given by volunteers. We can finalize the plans once the seminar gets going. The seminar will meet once a week, for a 90 minutes session.

Reading list. To begin with, the aim is to cover the material in the first 100 pages of the book ‘Champs Algébriques’ by Laumon and Moret-Bailly. The new textbook ‘Algebraic Spaces and Algebraic Stacks’ by Olsson will also be used. The online ‘Stacks project’ is a very useful reference, though too difficult to read as a textbook.