Title: Sub-convexity problems: Some history and recent developments

Abstract:

Bounding automorphic L-functions on the critical line $\mathrm{Re}(s) = 1/2$ is a central problem in the analytic theory of L-functions. The functional equation and the Phragmen-Lindelöf principle from complex analysis yield the convexity bound $L(1/2+it,\pi) \ll C(\pi,t)^{1/4+\varepsilon}$ where $C(\pi,t)$ is the "analytic conductor" of the L-function. Lindelöf hypothesis, which is a consequence of the Grand Riemann Hypothesis (GRH), predicts that the bound $C(\pi,t)^{\varepsilon}$ for any $\varepsilon > 0$. Any bound with exponent smaller than $1/4$ is called a sub-convexity bound. In this context the Weyl exponent $1/6$, which is one-third of the way down from convexity towards Lindelöf, is a known barrier which has been achieved only for a handful of families. First sub-convexity bound is proved by Hardy-Littlewood and Weyl independently for the Riemann zeta function.

In this talk we shall talk about some recent developments and new techniques. This talk is meant for a general audience and we shall be explicitly defining the relevant terms.