Solving integro-differential problems with Lanczos' spectral Tau method

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The Tau method is a spectral method, originally developed by Cornelius Lanczos, that delivers polynomial approximations to the solution of differential problems. The method tackles both initial and boundary value problems with ease. It is a spectral method thus ensuring excellent error properties, whenever the solution is smooth.

Initially developed for linear differential problems with polynomial coefficients, it has been used to solve broader mathematical formulations: functional coefficients, nonlinear differential and integro-differential (systems of) equations. Nevertheless, the Tau method has never been offered as a general purpose numerical tool.

In this presentation, the Lanczos' Tau method is examined in detail from a variety of aspects to provide a stable implementation for its operational version. We concentrate on avoiding basis transformation, on performing polynomial evaluations directly on the orthogonal basis, on tackling nonlinear problems and how to effectively compute polynomial approximations from non-polynomial coefficient functions.

The ultimate goal is to deploy a robust and efficient numerical library, the `Tau Toolbox`, able to deliver approximate solutions of integro-differential problems.

References

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