Venue: Room 216

ON MOMENT PROBLEMS - HISTORICAL ORIGINS, SIGNIFICANCE AND RECENT DEVELOPMENTS

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ABSTRACT. The classical moment problem for the half-line and the real line were formulated and solved by Stieltjes (1894) and Hamburger (1920) respectively, and the theory has been treated in influential monographs by Shohat-Tamarkin (1943) and Akhiezer (1965). The refined theory of the indeterminate moment problem, i.e., the case where two and hence infinitely many different measures have the same moments, is a delicate blend of complex function theory and spectral theory. It has gained renewed interest with the study of orthogonal polynomials associated with q-basic hypergeometric functions important for the representation theory of quantum groups, because many of these new orthogonal polynomials come from indeterminate moment problems.

Also several remarkable formulas of Ramanujan can be understood in the light of indeterminate moment problems.

It is only during the last two decades that one has been able to make rather complete calculations of all the relevant "characters" appearing in a concrete indeterminate moment problem.

To be specific let μ be a probability distribution with moments of any order

$$s_n = \int x^n \, d\mu(x), \quad n = 0, 1, \dots, \tag{1}$$

and consider the orthonormal polynomials P_n, i.e.,

$$\int P_n(x) P_m(x) \, d\mu(x) = \delta_{nm}.$$

They satisfy $P^2(z) := \sum |P_n(z)|^2 < \infty$ for all complex z precisely in the indeterminate case. This leads to a study of entire functions like

$$K(z, w) = \sum_{n=0}^{\infty} P_n(z) P_n(w), \quad z, w \in \mathbb{C}$$

and

$$L(z) = \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{s_{2n}}}.$$

The log-normal distribution is an example of an indeterminate measure.

We will give a review of the theory together with new results about the relation between the growth of P and summability properties of the sequence $(P_n(z))$. The order of the function P is called the order of the moment problem.

The multidimensional moment problem will also be discussed.