Venue: Room 216

ON THE DETERMINACY/INDETERMINACY OF THE STIELTJES MOMENT SEQUENCES $(n!)^c, c > 0$

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Abstract. Let f be a non-zero Bernstein function, i.e.,

$$f(s) = a + bs + \int_0^\infty \left(1 - e^{-xs}\right)\, d\nu(x),\ a, b \geq 0, \nu \geq 0.$$

There exists a uniquely determined product convolution semigroup $(\rho_c)_c > 0$ on $(0, \infty)$ such that

$$\int_0^\infty x^n\, d\rho_c(x) = (f(1) \cdot \cdots \cdot \cdot f(n))^c, \quad c > 0, n = 0, 1, \ldots, \tag{1}$$

see [1]. The Stieltjes moment sequence in (1) is always determinate when $c \leq 2$ as an easy consequence of the Carleman criterion. However, for $c > 2$, it can be determinate or indeterminate depending on f. In fact, in the case $f(s) = s$, where the moment sequence is $(n!)^c$, it was proved in [1] that the moment sequence is indeterminate. In this case $\rho_c = e_c(t)dm(t)$, where m is Lebesgue measure on the half-line and

$$e_c(t) = \frac{1}{2\pi} \int_{-\infty}^\infty t^{ix-1} \Gamma(1 - ix)^c\, dx, \quad t > 0. \tag{2}$$

The proof of the indeterminacy was quite delicate based on asymptotic formulas for stable distributions due to Skorokhod.

In recent work with José López, Spain, [2] we have found the asymptotic behaviour of e_c at infinity, viz.

$$e_c(t) \sim \frac{(2\pi)^{(c-1)/2}}{\sqrt{c}} \frac{e^{-ct^{1/c}}}{t^{(c-1)/(2c)}}. \tag{3}$$

From (3) it is easy to derive the indeterminacy of e_c from a criterion of Krein. In the lecture I will give the necessary background for the above.

References

[1] Berg, C., On powers of Stieltjes moment sequences, I. *J. Theor. Prob.* **18** (2005), 871–889.

[2] Berg, C., López, J. L., Asymptotic behaviour of the Urbanik semigroup. To appear in J. Approx. Theory 2015.