Contractivity and complete contractivity for the finite dimensional Banach Spaces

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It is known that contractive homomorphisms of the disc and the bi-disc algebra to the space of bounded linear operators on a Hilbert space are completely contractive, thanks to the dilation theorems of B. Sz.-Nagy and Ando respectively. Examples of contractive homomorphisms of the (Euclidean) ball algebra which are not completely contractive was given by G. Misra. From the work of V. Paulsen and E. Ricard, it follows that if $m \geq 3$ and $\mathbb{B}$ is any ball in $\mathbb{C}^m$ with respect to some norm, then there exists a contractive linear map which is not complete contractive. The characterization of those balls in $\mathbb{C}^2$ for which contractive linear maps are always completely contractive remained open. In this talk, we intend to answer this question for balls $\Omega_{\mathbf{A}}$ in $\mathbb{C}^2$ which are of the form

$$\Omega_{\mathbf{A}} = \{z = (z_1, z_2) : \|z\|_{\mathbf{A}} = \|z_1 A_1 + z_2 A_2\|_{\rm op} \leq 1\}$$

for some choice of an 2-tuple of 2×2 linearly independent matrices $\mathbf{A} = (A_1, A_2)$.