**Title**: Multiple Zeta Values: a Survey

**Abstract**: L. Euler (1707–1783) investigated the values of the numbers
\[
\zeta(s) = \sum_{n \geq 1} \frac{1}{n^s}
\]
for \(s\) a rational integer, and B. Riemann (1826–1866) extended this function to complex values of \(s\). For \(s\) a positive even integer, \(\zeta(s)/\pi^s\) is a rational number. Our knowledge on the values of \(\zeta(s)\) for \(s\) a positive odd integer is extremely limited. Recent progress involves the wider set of numbers
\[
\zeta(s_1, \ldots, s_k) = \sum_{n_1 > n_2 > \cdots > n_k \geq 1} \frac{1}{n_1^{s_1} \cdots n_k^{s_k}}
\]
for \(s_1, \ldots, s_k\) positive integers with \(s_1 \geq 2\).
Department of Mathematics
Indian Institute of Technology Bombay

Department Colloquium

Speaker:
Prof. Alex Gorodnik
Department of Mathematics
University of Bristol
Bristol, UK

Title: Diophantine Approximation by Orbits

Abstract:
The classical theory of Diophantine approximation quantifies the density of rational number in the real line. In a joint work with A. Ghosh and A. Nevo we consider an analogous problem of approximating by dense orbits for group actions. We explain a general approach which allows to establish quantitative density and gives the best possible exponents of approximation in a number of cases.

Date: Wednesday, November 19, 2014
Time: 16:00 - 17:00
Venue: Ramanujan Hall, Dept. of Mathematics
Title: Algebraic Independence Theory, an Overview

Abstract: The purpose is to present some results of algebraic independence of classical numbers that have been obtained by methods of diophantine approximation. Conjectures and open problems will be mentioned as well as the main tools used in the proofs.

Date: Wednesday, November 12, 2014
Time: 16:00 - 17:00
Venue: Ramanujan Hall, Dept. of Mathematics
Title: On an Equivalence of Derived Categories

Abstract:
We will describe a trick on Koszul complexes allowing us to prove that for a Cohen-Macaulay ring, there is an equivalence between the bounded derived categories of certain resolving subcategories. We can then apply this result to various generalized cohomology theories. This is joint work with William Sanders, growing out of earlier work with Satya Mandal.

Date: Wednesday, November 05, 2014
Time: 16:00 - 17:00
Venue: Ramanujan Hall, Dept. of Mathematics
Abstract:
Consider the following simple problem: given a set of linear inequalities, decide whether it has a feasible solution or prove that there is no feasible solution for it. How easy or hard is this problem? How does one quantify the easiness or hardness of this problem? Such questions are central to the field of proof complexity.

In this talk, we will give a brief introduction to the field of proof complexity. This is the area of theoretical computer science which classifies mathematical statements based on the lengths of the proofs required to prove them and the resources required to verify the correctness of the alleged proofs.

We will start the talk with a few motivating examples. We will then introduce the notion of a proof system, which allows us to perform a systematic study of proofs. We will look at different types of proof systems and give a list of known results. We will end the talk with a list of open problems.
Special Colloquium

Speaker:

Prof. Madhu Sudan  
Principal Researcher  
Microsoft Research New England  
Cambridge, USA

Title: Limits of Local Algorithms Over Sparse Random Graphs

Abstract: Local algorithms on graphs are algorithms that run in parallel on the nodes of a graph to compute some global structural feature of the graph. Such algorithms use only local information available at nodes to determine local aspects of the global structure, while also potentially using some randomness. Recent research has shown that such algorithms show significant promise in computing structures like large independent sets in graphs locally. Indeed the promise led to a conjecture by Hatami, Lovasz, and Szegedy, that local algorithms may be able to compute maximum independent sets in (sparse) random $d$-regular graphs. In this talk we refute this conjecture and show that every independent set produced by local algorithms is multiplicative factor $1/2 + 1/(2\sqrt{2})$ smaller than the largest, asymptotically as $d \to \infty$. Our result is based on an important clustering phenomena predicted first in the literature on spin glasses, and recently proved rigorously for a variety of constraint satisfaction problems on random graphs. Such properties suggest that the geometry of the solution space can be quite intricate. The specific clustering property, that we prove and apply in this paper shows that typically two large independent sets in a random graph either have a significant intersection, or have a nearly empty intersection. As a result, large independent sets are clustered according to the proximity to each other. While the clustering property was postulated earlier as an obstruction for the success of local algorithms, such as for example, the Belief Propagation algorithm, our result is the first one where the clustering property is used to formally prove limits on local algorithms. Based on joint work with David Gamarnik.

Date: Tuesday, October 07, 2014  
Time: 16:00 - 17:00  
Venue: Ramanujan Hall, Dept. of Mathematics
Consider a variety given by simultaneous vanishing of two quadratic forms in $n$ many variables. According to a conjecture of Manin such a variety should have "lots" of rational points if $n > 4$. This was proved by Birch for $n > 13$ in 1962. Recently this was improved and the right asymptotic was established for $n > 10$. This will be the main focus of the talk.
Speaker:
Prof. Muthu Krishnamurthy
Department of Mathematics
University of Iowa
Iowa City, USA

Title:
Converse Theorems

Abstract:
I will discuss my recent work with Andy Booker on Converse Theorems. A typical converse theorem gives a characterization of modular forms, or more generally (higher rank) automorphic forms, in terms of analytic properties of various twisted $L$-functions associated to them. Besides their applications in the Langlands program, they validate Langlands’ philosophy that automorphic forms are the right source for $L$-functions with nice analytic properties. Over the years, many people have worked on reducing the size of the twisting set in converse theorems. We take a slightly different approach wherein we try to reduce the analytic properties required of the twists rather than the twisting set itself.

Date : Wednesday, September 24, 2014
Time : 16:00 - 17:00
Venue: Ramanujan Hall, Dept. of Mathematics
Speaker:

Prof. Baskar Balasubramanyam
Department of Mathematics
IISER Pune
Pune, INDIA

Title:

Special Values of Adjoint L-functions and Congruences Between Automorphic Forms

Abstract:

Let $f$ and $g$ be primitive cusp forms with Fourier coefficients $a_n(f)$ and $a_n(g)$; $f$ and $g$ are said to be congruent modulo a prime $p$ if $a_n(f) = a_n(g) \mod p$. I will give an overview of results that relate this phenomenon to the value at $s = 1$ of a certain $L$-function attached to the cusp form $f$. I will then talk about my joint work with A. Raghuram that generalizes these results.

Date : Wednesday, September 17, 2014
Time : 16:00 - 17:00
Venue : Ramanujan Hall, Dept. of Mathematics
Homotopy Groups and Closed Geodesics on 4-manifolds

Abstract:
A simply connected, closed four manifold, is associated to a simply connected, closed, spin five manifold which occurs as the total space of a circle bundle. This leads to several consequences: the homotopy groups of such a four manifold are determined by its second Betti number, and the ranks of the homotopy groups can be explicitly calculated. We also show that for a generic metric on such a smooth four manifold with second Betti number at least three the number of geometrically distinct periodic geodesics of length at most $l$ grows exponentially as a function of $l$. 
We discuss the method of detecting exotic structures on $K$-hyperbolic manifolds given by F.T. Farrell, L.E. Jones and C.S. Aravinda, where $K = \mathbb{R}, \mathbb{C}, \mathbb{H}$ or $\mathbb{O}$. We also discuss the following question raised by F.T. Farrell and C.S. Aravinda (2004).

**Question:** For each division algebra $K$ over the reals and each integer $n \geq 2$ ($n = 2$ when $K = \mathbb{O}$), does there exist a closed negatively curved Riemannian manifold $M^{dn}$ (where $d = \dim_{\mathbb{R}} K$) which is homeomorphic but not diffeomorphic to a $K$-hyperbolic manifold?

Finally, we give some observations on the work of F.T. Farrell, L.E. Jones, C.S. Aravinda and C.T.C. Wall.
Mathew Kahle and Elizabeth Meckes recently established interesting results concerning the topology of the clique complex $X(n, p)$ on an Erdos Renyi graph $G(n, p)$ when $p = n^\alpha$ with $\alpha \in (-1/k, -1/(k + 1))$. They showed that as the number of vertices $n$ tends to infinity, every Betti number $\beta_j$ of $X(n, p)$ vanishes to zero except for the $k$-th one. In fact, $\beta_k$ follows a central limit theorem, i.e., $(\beta_k - \mathbb{E}[\beta_k])/\sqrt{\text{Var}(\beta_k)}$ is asymptotically Gaussian.

In this talk, we will extend the above result to the case of a randomly evolving Erdos Renyi graph $G(n, p, t)$. We will show that if $p$ is chosen as above, then the process $(\beta_k(t) - \mathbb{E}[\beta_k(t)])/\sqrt{\text{Var}[\beta_k(t)]}$ is asymptotically an Ornstein-Uhlenbeck process. That is, the $k$-th Betti number asymptotically behaves like a stationary Gaussian Markov process with an exponentially decaying covariance function.

This is joint work with Prof. Robert Adler.
Cohen-Macaulay Rings of Finite Representation Type

Abstract:
In this talk I will discuss complete Cohen-Macaulay local rings of finite representation type, i.e., it only has finitely many isomorphism classes of indecomposable maximal CM modules. This theory connects commutative algebra with representation theory and with the theory of Arnold’s simple singularities.
In 1916, in a little known paper, Ramanujan introduced the idea of a “Fourier series” for arithmetical functions. It is clear that these ideas were in Ramanujan’s mind before he left for England in 1914 since some of these notions are hinted at in his letters to Hardy written earlier. In this talk, I will give a general outline of his ideas and how they can be used to understand problems such as the twin prime conjecture.
Regenerative Sequence Monte Carlo Methods for Improper Target Distributions

Abstract:
If \( \pi \) is a probability distribution on some set \( S \) and \( f \) is a real valued function with \( E|f(X)| < \infty \) where \( X \) is a random variable with distribution \( \pi \) then the parameter \( \lambda = Ef(X) \) can be estimated using either IID Monte Carlo (IIDMC) or Markov chain Monte Carlo (MCMC) methods. In this talk we describe a new monte carlo procedure called RSMC (Regenerative sequence monte carlo) for this problem and show it works for the case whether \( \pi \) is proper, i.e, \( \pi(S) < \infty \) or not proper, i.e, \( \pi(S) = \infty \). We illustrate this with estimating a convergent infinite series and an integral over \( \mathbb{R} \) with respect to Lebesgue measure. We also provide a confidence interval for \( \lambda \).
Department of Mathematics
Indian Institute of Technology Bombay

Department Colloquium

Speaker:
Prof. Dinesh Thakur
Department of Mathematics
University of Rochester
Rochester, NY, USA

Title:
Congruences, Derivatives and Euler constant

Abstract:
We will describe how some fundamental questions on congruences modulo powers of primes connect to derivatives and to Euler’s constant.

Date : Wednesday, July 30, 2014
Time : 16:00 - 17:00
Venue: Ramanujan Hall, Dept. of Mathematics
Computing Teichmüller Maps Between Polygons

Abstract:
By the Riemann mapping theorem, one can bijectively map the interior of an $n$-gon $P$ to that of another $n$-gon conformally. However, (the boundary extension of) this mapping need not necessarily map the vertices of $P$ to those of $Q$. In this case, one wants to find the “best” mapping between these polygons; i.e., one that minimizes the maximum angle distortion (the dilatation) over all points in $P$. I shall discuss this problem in the continuous and the discrete settings. This topic lies on the interface of complex analysis and theoretical computer science.
Suppose that \( s \) students want to equally share \( c \) cakes. What is the smallest number of cake pieces, \( p(c, s) \), needed to achieve this fair distribution? We will derive a formula for \( p(c, s) \) and describe two different distribution schemes that achieve this. One of them is associated with a square tiling of a \( c \times s \) rectangle \( R \), and we shall see that this square tiling is “isoperimetric” in the sense that it has smallest “perimeter” among all square tilings of \( R \). I will describe a generalized version of this problem that is still open.