Bisection closed and bisection free families

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(joint work with Prof. Niranjan Balachandran and Prof. Rogers Mathew)

Let n be any positive integer and $\mathcal{A}$ be a family of even subsets of $[n]$. A set B *bisects* another set A if $|A \cap B| = \frac{|A|}{2}$. Given a family $\mathcal{A}$, another family $\mathcal{B}$ of subsets of $[n]$ is called a *bisecting family* for $\mathcal{A}$, if for each subset $A \in \mathcal{A}$, there exists a subset $B \in \mathcal{B}$ that bisects A. This problem has been studied in detail in [1]. We consider the following extension of the notion of bisection. A family $\mathcal{A}$ consisting of even subsets of $[n]$ is called *bisection closed* if for each $A, B \in \mathcal{A}$, either A bisects B or B bisects A (or both). Let $\vartheta(n)$ ($\vartheta(n,k)$) denote the maximum cardinality of any (respectively, a k-uniform) *bisection closed* family on $[n]$. A family $\mathcal{F}$ of subsets of $[n]$ is called *bisection free* if for each subset $A \in \mathcal{F}$, there is no subset $B \in \mathcal{F}$ that bisects A. This can be viewed as an extension of l-avoiding families [2, 3]: a family $\mathcal{F}$ is called l-avoiding if $|A \cap B| \neq l$ for some $l \in [n]$. Let $\bar{\vartheta}(n)$ denote the maximum cardinality of a bisection free family on $[n]$. In the talk, we would discuss few interesting results on $\vartheta(n)$ and $\bar{\vartheta}(n)$ and discuss the open problems.

Speaker Bio Tapas Kumar Mishra received a Ph.D. Degree in Computer Science & Engineering from Indian Institute of Technology, Kharagpur in Sept, 2017 under the supervision of Prof. Sudebkumar Prasant Pal and Prof. Rogers Mathews. He received a M. Tech. and B.Tech Degrees in Computer Science & Engineering from Indian Institute of Technology, Kharagpur and Veer Surendra Sai University of Technology, Burla in 2013 and 2010, respectively. His research interests are Combinatorics, Graph Theory and Approximation algorithms.

References

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