

## Primes in sumsets and sumsets of primes.

### Abstract

In 2014, Ramana and Ramaré showed that if the set of prime numbers is coloured in  $K \geq 1$  colours then all large enough integers are a sum of at most  $CK \log \log 4K$  prime numbers, all of the same colour, for an absolute constant  $C$ . This bound is optimal up to the value of  $C$ . Recently, K. Malleham adapted the method of Ramana and Ramaré to obtain an upper bound for the number of pairs  $(a, b) \in A \times B$  such that  $a + b$  is a prime number, when  $A$  and  $B$  are subsets of the integers  $[1, N]$  and  $N$  is sufficiently large. The bound obtained by Malleham improves a bound obtained by Balog, Rivat and Sárközy and is also optimal when  $|A||B|$  is sufficiently large.

Underlying the aforementioned results is an upper bound for the number of pairs  $(a, b) \in A \times B$  such that  $a + b$  is invertible in  $\mathbf{Z}/W\mathbf{Z}$ , where  $W$  is a suitable square free number and  $A$  and  $B$  certain subsets of the integers. We discuss this result and some of its relatives and explain how they may be used to obtain analogs of the result of Ramana and Ramaré for the set of squares and the set of squares of prime numbers.