L^p-Theory for the Stokes and Navier-Stokes Equations with Different Boundary Conditions

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Abstract. We consider here elliptical systems as Stokes and Navier-Stokes problems in a bounded domain, eventually multiply connected, whose boundary consists of multi-connected components. We investigate the solvability in L^p theory, with $1 < p < \infty$, under the non standard boundary conditions

$$\boldsymbol{u} \cdot \boldsymbol{n} = g, \quad \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{h} \quad \text{or} \quad \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{g}, \quad \pi = \pi_* \quad \text{on}\, \Gamma.$$

We consider also the case of Navier boundary conditions:

$$\boldsymbol{u} \cdot \boldsymbol{n} = g \quad \text{and} \quad 2\left[\mathbf{D}(\boldsymbol{u})\boldsymbol{n}\right]_{\boldsymbol{\tau}} + \alpha \boldsymbol{u}_{\boldsymbol{\tau}} = \boldsymbol{h} \quad \text{on}\, \Gamma$$

where α is a friction coefficient and $\mathbf{D}(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^\top)$ is the stress tenseur. The main ingredients for this solvability are given by the Inf-Sup conditions, some Sobolev's inequalities for vector fields and the theory of vector potentials satisfying

$$\boldsymbol{\psi} \cdot \boldsymbol{n} = 0, \quad \text{or} \quad \boldsymbol{\psi} \times \boldsymbol{n} = \mathbf{0} \quad \text{on}\, \Gamma.$$

Those inequalities play a fundamental key and are obtained thanks to Calderon-Zygmund inequalities and integral representations. In the study of ellpitical problems, we consider both generalized solutions and strong solutions that very weak solutions.

In a second part, we will consider the nonstationary case for the Stokes equations.

References.

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