A study of matrices associated with complex unit gain graphs

Aniruddha Samanta

In this talk, we study complex unit gain graphs via the spectra of various classes of matrices associated with them. A complex unit gain graph ($\mathbb{T}$-gain graph), $\Phi = (G, \varphi)$ is a graph where the function φ assigns a unit complex number to each orientation of an edge of G, and its inverse is assigned to the opposite orientation. We study several spectral properties of $\mathbb{T}$-gain graphs. In particular, we characterize the bipartite graph in terms of gains. Then, we study the cospectrality of $\mathbb{T}$-gain graphs. Besides, we provide many bounds for eigenvalues of Φ in terms of the number of vertices, number of edges, degree of vertices, etc.

Next, we establish bounds for the spectral radius of $\mathbb{T}$-gain graphs in terms of largest eigenvalues and largest vertex degree and identify classes of graphs and gains for which the inequality is sharp. We introduce k-generalized Hermitian adjacency matrix of a mixed graph X and characterize the structure of X for which $\rho(H_k(X)) = \Delta$ holds.

Then, we focus on the bounds of energy of $\mathbb{T}$-gain graphs in terms of vertex cover number, largest vertex degree, smallest vertex degree, etc. Particularly, we establish $2\tau - 2c \leq \mathcal{E}(\Phi) \leq 2\tau\sqrt{\Delta}$ and characterize both the equalities, where τ and c are the vertex cover number and the number of odd cycles of G, respectively. The characterization completely solves an open problem in a more general setting. For any triangle-free $\mathbb{T}$-gain graph Φ, we prove that $\mathcal{E}(\Phi) \geq 2\delta$, where δ is the minimum vertex degree. In addition, a number of bounds for the energy of $\mathbb{T}$-gain graphs are obtained in terms of the vertex degree, edge degree, spectral radius, etc.

Finally, we propose two notions of gain distance matrices $\mathcal{D}^{\max}_{<}(\Phi)$ and $\mathcal{D}^{\min}_{<}(\Phi)$ of a $\mathbb{T}$-gain graph Φ, for any ordering '$<$' of the vertex set and study their various properties. We call a $\mathbb{T}$-gain graph Φ is distance compatible iff $\mathcal{D}^{\max}_{<}(\Phi) = \mathcal{D}^{\min}_{<}(\Phi)$. Then, we characterize the distance compatible gain graphs. Besides, we introduce the notion of positively weighted $\mathbb{T}$-gain graphs and establish an equivalent condition for the balance of a $\mathbb{T}$-gain graph. Acharya's and Stanić's spectral criteria for balance are deduced as a consequence. Apart from the above thesis work, we study the multiplicity of A_α-eigenvalues of $\mathbb{T}$-gain graphs and improve an existing bound. Then, we study gain distance Laplacian matrices and extremal graph energy for $\mathbb{T}$-gain graphs.