Finite volume element method for subdiffusion problems

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Abstract

We consider a finite volume element method for approximating the solution of a time fractional diffusion problem involving a Riemann-Liouville time fractional derivative of order $\alpha \in (0, 1)$. For the spatially semidiscrete problem, we establish optimal with respect to the data regularity $L^2(\Omega)$-norm error estimates, for the cases of smooth and middly smooth initial data, i.e., $v \in H^2(\Omega) \cap H_0^1(\Omega)$ and $v \in H_0^1(\Omega)$. For nonsmooth data $v \in L^2(\Omega)$, the optimal $L^2(\Omega)$-norm estimate is shown to hold only under an additional assumption on the triangulation, which is known to be satisfied for symmetric triangulations. Superconvergence result is also proved and as a consequence a quasi-optimal error estimate is established in the $L^\infty(\Omega)$-norm. Further, two fully discrete schemes based on convolution quadrature in time generated by the backward Euler and the second-order backward difference methods are developed, and error estimates are derived for both smooth and nonsmooth initial data. Finally, some numerical results are presented to illustrate the theoretical results.

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