Glimpses of KdV Equation and Soliton Theory

Contributions from Observation, Physical & Numerical Experiments,

and Pure & Applied Mathematics

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Abstract

Solitons are solutions of a special class of nonlinear partial differential equations (soliton equations, the best example is the KdV equation). They are waves but behave like particles. The term "soliton" combining the beginning of the word "solitary" with ending "on" means a concept of a fundamental particle like "proton" or "electron".

The events: (1) sighting, by chance, of a great wave of translation, "solitary wave", in 1834 by Scott-Russell, (2) derivation of KdV E by Korteweg de Vries in 1895, (3) observation of a very special type of wave interactions in numerical experiments by Krushkal and Zabusky in 1965, (4) development of the inverse scattering method for solving initial value problems by Gardener, Greene, Kruskal and Miura in 1967, (5) formulation of a general theory in 1968 by P. D. Lax and (5) contributions to deep theories starting from the work by R. Hirota (1971-74) and David Mumford (1978-79), which also gave simple methods of solutions of soliton equations, led to the development of one of most important areas of mathematics in 20th century.

This also led to a valuable application of solitons to physics, engineering and technology.

There are two aspects soliton theory arising out of KdV Equation

- Applied mathematics - analysis of nonlinear PDE leading to dynamics of waves.
- Pure mathematics - algebraic geometry.

It is surprising that each one of these can inform us of the other in the intersection that is soliton theory, an outcome of KdV equation.
The subject too big but I shall try to give some glimpses (1) of the history, (2) of the inverse scattering method and (2) show that algorithm based on algebraic-geometric approach is much easier to derive soliton solutions.