Speaker

Krishna B. Athreya
Professor Emeritus, Department of Mathematics, Iowa State University

Title : **Some results in probability theory with application to analysis**

Abstract

In this talk we shall prove the following results in probability theory and give one application to analysis of each of the results.

Result 1: Let f_n, f be probability densities on a measure space $(\Omega, \mathcal{B}, \mu)$, i.e. $f, f_n : \Omega \to \mathbb{R}^+$, $\mathcal{B}$ measurable and

$$\int f_n d\mu = 1 \text{ for all } n \text{ and } \int f d\mu = 1.$$

Then

$$\int |f_n - f| d\mu \to 0 \text{ as } n \to \infty \text{ iff } f_n \to f \text{ in measure } \mu$$

i.e. $\forall\ \epsilon > 0,\ \mu\{\omega : |f_n(\omega) - f(\omega)| > \epsilon\} \to 0 \quad \text{as } n \to \infty.$

Application: $\exists \mu_n, \mu$ probability measures on a measurable space $(\Omega, \mathcal{B}) \ni \mu_n(A) \to \mu(A)\ \ \forall\ \ A \in \mathcal{B}$ but $\sup\limits_{A \in \mathcal{B}} |\mu_n(A) - \mu(A)| \not\to 0$ as $n \to \infty$.

Result 2: Let $\{X_i\}_{i \geq 1}$ be iid Poisson (1) random variables. Then

$$\sqrt{n} P(S_n = n) \to \frac{1}{\sqrt{2\pi}} \text{ as } n \to \infty$$

where $S_n = \sum_{j=1}^n X_j$.

Application: $n! \sim \dfrac{e^{-n} n^{n+\frac{1}{2}}}{\sqrt{2\pi}}$ as $n \to \infty$.

Result 3: Let $\{X_i\}_{i \geq 1}$ be i.i.d. random variables with finite mean μ. Then $\forall\ \epsilon > 0,\ P(|\overline{X}_n - \mu| > \epsilon) \to 0$ as $n \to \infty$.

Application: Let $f = [0, 1] \to \mathbb{R}$ be continuous. Then $\forall\ n > 0,\ \exists$ a polynomial $p_n(x)$ such that

$$\sup_{0 \leq x \leq 1} |f(x) - p_n(x)| \to 0 \text{ as } n \to \infty.$$