AVERAGING AND INITIAL LAYER ANALYSIS IN PASSIVE TRANSPORT

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Abstract. Passive transport models are equations of advection-diffusion type. In most of the applications involving passive transport, the advective fields are of greater magnitude compared to molecular diffusion. This talk attempts to present a novel theory developed by myself, Thomas Holding (Imperial) and Jeffrey Rauch (Michigan) to address these strong advection problems. Loosely speaking, our strategy is to recast the advection-diffusion equation in moving coordinates dictated by the flow associated with the advective field. Crucial to our analysis is the introduction of a fast time variable and the introduction of some new notions of weak convergence along flows in L^p spaces. We also use ideas from the theory of "homogenization structures" developed by Gabriel Nguetseng.

Our asymptotic results show the following dichotomy:

- If the Jacobian matrix associated with the flow satisfies certain structural conditions (loosely speaking, boundedness in the fast time variable) then the strong advection limit is a non-degenerate diffusion when seen along flows.
- On the other hand, when the Jacobian matrix associated with the flow fails to satisfy the aforementioned structural conditions, then the strong advection limit is a parabolic problem with a constraint. Here we show the appearance of an initial layer where there is an enhanced dissipation along flows.

Our results have close links to

- the Freidlin-Wentzell theory on perturbations of dynamical systems.
- the theory of Relaxation enhancing Lipschitz flows.

This talk will illustrate the theoretical results via various interesting examples. We address some well-known advective fields such as the Euclidean motions, the Taylor-Green cellular flows, the cat's eye flows and some special class of the Arnold-Beltrami-Childress (ABC) flows. We will also comment on certain examples of hyperbolic or Anosov flows.

Some of the results to be presented in this talk can be found in the following publication:

T. Holding, H. Hutridurga, J. Rauch. Convergence along mean flows, SIAM J Math. Anal., Volume 49, Issue 1, pp. 222–271 (2017).

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