Department of Mathematics, Indian Institute of Technology Bombay
MA522: Fourier Analysis and Applications

Course Information

S. Sivaji-Ganesh Spring 2011

(1) **Classes:** Classes will be held in Room no. 105, Mathematics department in Slots 5A, 5B, XD. The slot 5A will be used for Tutorials and Quizzes. In addition to these three classes, I will take another class (this will be **optional**) whose timing will be fixed soon; this will be used to address your questions/doubts in the assignments. You can see me in my office for any assistance either when I am available in my office or by taking an appointment by an e-mail.

(2) **Prerequisites:** The prerequisite for this course is **MA 403: Real Analysis I**. It is assumed that you are familiar and comfortable working with concepts like Compactness, Uniform convergence of functions. At times we will be using Lebesgue spaces like square integrable measurable functions, and also some convergence theorems like Dominated convergence theorem amd Monotone convergence theorem. I will recall all relevant results from these topics whenever they are needed and will provide references to texts where they are discussed in depth; make sure that you are comfortable working with them.

(3) **Detailed Syllabus:** The syllabus consists of three parts. Detailed syllabus is given below in the order of coverage.

(a) **Distributions and Fourier Transforms:** Distributions - Operations on distributions: Derivatives, Multiplication with functions, Convolutions and their characterization- Order and Support of a distribution - Fundamental solutions.

Fourier transform - Schwartz's class of rapidly decreasing functions - Characterization of fourier transform - Paley-Wiener theorems.

(b) **Fourier Series:** Summability of Fourier series - Abel, Cesàro, mean square, pointwise convergences - Dirichlet, Fejér kernels.

(c) **Applications:** Peetre's theorem - Fundamental solutions of the three basic partial differential equations and their properties - Weyl's equidistribution theorem - Shannon's sampling theorem - Collapsing bridges - Tomography. (a selection of these depending on availability of time)

(4) **Texts/References:**

Chief text There are many books written on Fourier analysis; most of which are aimed at an advanced student of mathematics. However there are also books that introduce the same material at a leisurely pace. One such book is

J.J. Duistermaat and J.A.C. Kolk, *Distributions: Theory and applications*, (Birkhauser, 2008),

and we will study the first 16 chapters of this book. This text has problems of different difficulty levels. I will indicate in the class the topics and problems from the text book that are beyond the scope of the course and you can safely ignore them.

The following are some useful references.

References

(i) F.G. Friedlander and M. Joshi, *Introduction to the theory of distributions*, (Cambridge, 1998). This book treats the subject almost similar to our text but in a fewer number of pages.

(ii) S. Kesavan, *Topics in functional analysis and applications*, (New age international, 1989). Most of our syllabus is present in the first chapter.

(iii) E.M. Stein and R. Shakarchi, *Fourier analysis: An introduction*, (Princeton University Press, 2003). This book deals with mainly Fourier series.

(iv) R.S. Strichartz, *A guide to Distribution theory and Fourier transforms*, (World scientific, 2003).

(5) **Course Objectives:**

(a) This is a **rigorous mathematics course** and not a methods course. That is, we will neither compute Fourier transforms of hundreds of functions nor compute Fourier series of thousands of functions. You are expected to write rigorous proofs with clear mathematical details behind every step of the proof.

(b) You will be at the right place to start understanding generalized solutions of differential equations which are unavoidable due to the physical background of certain Partial differential equations.

(c) You would have seen methods of solving certain differential equations using Fourier series and transforms. After doing this course, you should be able to decide which of them can be mathematically justified.

(6) **Assignments:** I will give assignment sheets every week. They contain a long list of problems which define the mathematical level and scope of the course. Solutions to some of those problems have to be submit for evaluation by the due date. Late submissions may be accepted but with reduced weightage. We will discuss most of these problems in the tutorial hour and also in the special class intended for the same. You are expected to attempt all the problems before coming to the tutorial class, as there might be a weightage for the class participation.

(7) **Attendance:** Currently 80% attendance is compulsory failing which you will be awarded an **XX** grade. This requirement is **non-negotiable**. It is your duty to make sure that you are not falling below the required attendance levels.

(8) **Grading Scheme:** Your grade will depend on the performance in assignments and three to five Quizzes (40 marks), Mid-sem (20 marks), End-sem (40 marks). The highest mark will be placed at an appropriate grade and the rest will be graded relatively.

(9) **Copying in assignments and examinations:** Any student found copying will be reported for disciplinary action.

(10) **Course webpage:** A course webpage will be maintained and it can be accessed from http://www.math.iitb.ac.in/∼siva